The Ghost of Relativity and Modality in Quantum Physics

Abstract for Invited Presentation for "Physics Beyond Relativity 2019"

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1 Mathematical continuum v.s. physical continuum

1.1 Mathematical continuum

The term "continuum" has been used casually in theoretical physics, causing some alarming situations. We will briefly discuss what continuum really means mathematically so that physics will not step into some fundamental conceptual confusion in considering continuum structure.

An infinite set X is *countable* if we can list up its members as

$$X = \{x_0, x_1, x_2, ...\} = \{x_i : i \in N\}$$

where N is the set of all natural numbers.

Example 1 *1. The set E of all even numbers is countable, as the function $f : N \to E$ such that $f(n) = 2n$ lists up all even numbers. Similarly the set O of all odd numbers is countable. 2. The set of all rational numbers is countable. To show this we first remember that all rational numbers can be expressed as n/m where n and m are natural numbers and $m \neq 0$. Now we can list all rational*

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numbers as

$$
\begin{array}{ccccccc}
\frac{1}{1} & & \frac{1}{2} & \rightarrow & \frac{1}{3} & & \\
\downarrow & \nearrow & & \swarrow & & \nearrow & \\
\frac{2}{1} & & \frac{2}{2} & & \frac{2}{3} & & . \\
 & \swarrow & & \nearrow & & \swarrow & . \\
\frac{3}{1} & & \frac{3}{2} & & \frac{3}{3} & & . \\
\downarrow & \nearrow & & \swarrow & & \nearrow & . \\
 & & . & . & . & . &
\end{array}
$$

Cantor "hypothetically" listed all elements of an open interval $(0, 1)$ as follows:

$$
\begin{array}{l}
0.d_{11}d_{12}d_{13}, ... \\
0.d_{21}d_{22}d_{23}......... \\
....
\end{array}
$$

He created a new real number as $x = 0.d_1d_2d_3, ...$ such that $d_1 \neq d_{11}$, $d_2 \neq d_{22}$, $d_3 \neq d_{33}$, ... Clearly x is in $(0, 1)$. But it can not appear in the listing above at the pain of contradiction. So he rightly concluded that the set R of all real numbers is not countable. [**Proof by Contradiction**]

Indeed, as we discussed above, we can enumerate all rational numbers but we can *not* enumerate all irrational numbers. Indeed, we can show that almost all real numbers are irrational numbers using Weierstrass function which is defined over the interval $(0, 1)$ as

$$w(x) = \textbf{if } x \text{ is rational } \textbf{then } 0 \textbf{ else } 1$$

The Lebesque integral of this function over $(0, 1)$ is 1.

Any interval (a, b) of real numbers is called *"continuum"*. Clearly there are way more points in the continuum than in countable sets as we discussed above.

Interestingly, there are two kinds of countable sets of numbers. If we consider the set of integers, for each integer, there is a next larger integer. But if we consider the set of all rational numbers, in between any two rational numbers, we have a rational number in between. This means that there is no such thing as the next larger rational number for any rational number. Mathematicians say the set of all rational numbers with natural ordering is *dense*. It also is the case that the set of all real numbers with natural ordering is dense in the same sense. This is because the set of rational numbers and the set of real numbers both have total natural ordering which makes them dense. The natural ordering of natural numbers or integers do not make the set of natural number and that of integers dense. In summary, the natural ordering of rational (or real) numbers is dense ordering but the natural ordering of natural numbers (or integers) is *not* dense.

1.2 Physical continuum

It has been assumed as an "empirical common sense" that there are at most countably many (more likely finitely many) particles in the universe. This assumption comes into a conflict with the mathematical reality. We have been told that the wave frequency of electromagnetic waves *in theory* can be any positive real number. This means that there are "continuumly many frequencies" of electromagnetic waves. This implies, according to the relativistic theory of em waves as per Einstein, there are continuumly many particles called photons in this universe. The record shows that Planck disagreed with the idea of considering $h\nu$ a particle called photon. For him this was just a mathematical convention to deal with the blackbody radiation problem. This is a very good example of serious discrepancy between the concept of continuum in mathematics and that in theoretical physics.

Another manifestation of the discrepancy between the continuum for physics and that for mathematics can be seen in the fluid mechanics. In this theory, they consider a *"force applied to a unit area"*. They call it "pressure". In Newton dynamics, all physical bodies are reduced to point bodies and force as a vector ("pointed arrow") is applied to a "point body" not to a body with geometric dimension. So, purely "theoretically", *there is no such thing as applying force to a non-point body such as unit area or a "spaceship" contrary to the way Einstein thought.* This is treu "conceptually" too. More fundamentally, a unit surface is made of continuumly many geometric points and this leads to the assumption that just a unit area has uncountably infinite number of particles. This is a serious violation of the basic assumption on our universe where we assume that there are at most countably infinite particles.

Exactly the same thing happens in wave mechanics. For example when we consider the so called string waves, there are continuumly many geometric points in a string and each of them as a particle is supposed to be subjected to force. This is in conflict with the most fundamental assumption of dynamics that there are at most countably infinite number of atoms.

This is to say that the theory of particle dynamics and that of continuum dynamics are entirely different things. The very concept of motion and force in the particle dynamics and that in continuum dynamics are entirely different.

The calculus physicists use as a "language" is based upon the reasoning which contradicts the empirical expectation on the "number of particles".

Moreover, it is important to learn that *the set of infinitely many "particles" can never be dense.* So, the set of particles is very different from the continuum in this sense too. *This implies that there is no "particle" decomposition of continuum*, contrary to what quantum mechanists think. Topologists say that a point does not belong to the topological space. Aristotle said that a point is not a part of the line. Tragically the common response from "even theoretical" physicists is that "mathematics is just a language for physics."

It is tragic that physicists and mathematicians do not communicate anymore. Sensing that what is happening in theoretical physics is wrong mathematics, mathematicians do not pay attention to theoretical physics anymore. Physicists are not ready to accept any criticism from outside and sensing the view of mathematicians on their discipline, they say is "mathematics is just a language for physics." Under this situation, both mathematics and physics have been stagnating.

Remark 2 *What is tragic is this "split and indifference" between pure mathematics and theoretical (or mathematical) physics. Physicists say, mathematician's world is an irrelevant fantasy world. Mathematicians say physicists are "stupid", they do not understand the very mathematics they use. Newton was a single founder of modern mathematics (not like Euler's problem solving mathematics) and physics. For him these two worlds were intimately connected. Because of the intimate connection between physics and Capitalism in the form of engineering, physics community became political power. What Newton considered "philosophy" became the foundation of money making and dominating world politically. Certainly Leibniz was a philosopher too. On mathematics side, under social and political isolation, mathematics became "too pure" looking down upon the science which deals with the higher understanding of the real world issues. The ultimate achievement on this line is solving of long open problems Mathematics is reduced to problem solving. It was the Russian mathematician Perelman who solved the long open Poincare conjecture. He refused the offer of Fields award (Nobel Prize in mathematics) saying that this problem has not much importance to mathematics. He added that there are more important issues to consider in mathematics.*

1.3 What about quantum mechanics?

What we studied above asserts that it is not possible to "quantize" mathematical continuum. This issue has been unnoticed through out the development of Quantum Physics. By definition continuum is unquantizable What Dirac's work of quantizing em field and em waves showed us is the "first naive but elaborate approach" to the problem. But as an applied mathematician, he fell short of presenting material solution. The real issue is that he had not accurate grasp of the problem here. He did not understand the true nature of mathematical continuum as this was never taught in mathematical physics. *Though it is desirable and could be possible by some means, it requires a fundamental paradigm shift in pure mathematics to accommodate the needs of theoretical physics.* Since Newton and Leibniz it has been a tradition of physics to make this dialogue possible. Not any more. One hope here is that mathematics evolved into modern topology which could well offer some mathematically coherent "quantization of continuum" to meet the needs of physics. We do not think Witten's super string theory really addressed this important issue. It appears to be yet another fancy side tracking coming from ignoring physics.

Here is a list of some analysis of the current situation.

(1) It all started with Einstein's quantization of em waves. According to Einstein, for each frequency ν of the em wave, $h\nu$ is a particle called photon of frequency ν. Then as there are continuumly many frequencies ν of em waves,

there must be continuumly many particles called photons. But this is impossible as *no matter how we pack physical particles, we will never create a continuum.* Planck was careful and he did not call $h\nu$ a particle. As we discussed in our first paper "Logical Analysis of Relativity Theory" for the presentation in this Conference, Einstein's particle called photons lead to contradiction. They are paradoxical particles on their own physical behaviour too. This simply means that in reality, *there are no such thing as particles called photons*. So, Planck's reluctance to consider hv as a particle was justifiable.

(2) Dirac pushed this ill fated idea of Einstein's photons further without considering these fundamental issues and tried to quantize the entire (relativistic) electrodynamics, giving birth to a new field of physics called Quantum Electrodynamics (QED). In this theory, he considered the "space with energy" as an "infinite sea" of particles called photons. As the idea came from quantization of the harmonic oscillator through Fourier expansion, representing photons as the components of the expansion, there are only "countably many" photons which do not agree with two basic issues: *First*, Einstein-Planck's quantization of em waves assumes "uncountably infinite number of" photons, one for each frequency. So, Einstein's photons and Dirac's photons are not related. *Second*, as countably infinite collection, Dirac's view of photons can not satisfactory explicate the so called vacuum with energy.

Dirac could not realize these issues associated with his project. The regretful divorce between the King and the Queen made nobody care about this serious problem. On physics side the problem is its misunderstanding of what mathematical continuum is and on mathematics side, it is its misunderstanding of what physical continuum is meant to be. The history shows, instead of working together to resolve the problem, they just turned around walked away.

Remark 3 *In 1990's there was a movement to let physics and mathematics work together on shared problems. This was an example of the more global attempt to explore interdisciplinary areas. This good idea however produced very little as the focus was on shared mathematical problems at surface level. None of them really toughed the share problems at the most fundamental level as we have been discussing. Politically it was obviously the attempt to promote "research activities" such as "data mining" in which mathematical logicians and computer scientists worked together not to learn any deep truth but to create global monitoring public system.*

Going back to physics, the infamous problem of the infinite zero point energy in QED is a naturally expected consequence of this deficiency of QED. This problem is closely related to the correct view of Feynman who later concluded that it is the usage of the force field which causes the problem of divergence of energy in QED and recommended to go back to Weber-Gauss's action at a distance theory.

In the end, all of this should be the responsibility of philosophers of Physics and philosophers of Mathematics. These most prestigious institutions failed to provide any higher wisdom. All they did was to "thrive under the glory of physics and mathematics".

1.4 Infinitesimals in physics and in mathematics?

Conceptual difference between mathematical continuum and physical continuum which we briefly discussed at the beginning of this paper further leads to the issues of limit. Mathematically speaking, this problem is directly linked to the way physicists use calculus. "dx" in calculus means an "infinitesimal" which does not mean a very very small real number as physicists think. An infinitesimal means *a positive number which is smaller than any positive real number!* Naturally, such numbers are not real numbers which can be compared in magnitude with real numbers. The so called "calculus" which physicists use as a "language" is based upon this *apparently paradoxical concept* as Cantor complained. Newton, the founder of calculus was reluctant to use infinitesimals. Leibniz endorsed infinitesimals, though just like everybody else, he was not sure what it meant. It is quite clear that Leibniz preferred infinitesimals as the process of taking limit and calculating derivatives appear more like familiar algebra. One thing for sure is that *whatever infinitesimals are, they are not real numbers at the pain of contradiction.*

Cantor, the founder of set theory, categorically rejected this concept of infinitesimally for the reason above. Mathematical analysts (researchers of advanced calculus) avoided this mysterious concept altogether and following the advice of Cantor and Dedekind used the *topological concept of limit* to develop "precise calculus" which they called "mathematical analysis". *As mathematics this is perfect but we lost direct connection with physics.* Later, Newton's calculus which used naive limit concept was verified by the topological approach. But *limit is not an ontological process anymore.* We certainly needs more tight communication among theoretical physicists, experimental physicists and pure mathematicians. We would dare say that mathematical physicists should wait until the revision of mathematical physics is done in this way. Mathematical physicists were supposed to govern this complex activity of governing this defalcate but fundamental research activity. They tried to thrive on what was already done instead. It is just like the problem of philosophers.

It was Leibniz who used the "naive infinitesimals" to develop calculus which on surface is equivalent to Newton's calculus. Mathematicians avoided this approach for various different reasons. It was Abraham Robinson who developed a correct and complete theory of infinitesimals using the construction of "ultra power" which was developed by himself for a branch of mathematical logic called model theory.

Physicists have a naive version of infinitesimal. For them dx simply means a "very very small positive real number".There is no clear connection between Robinson's infinitesimal calculus and physicist's version of infinitesimal calculus. Indeed, it is clear that *a very small positive real number and a positive number which is smaller than all positive real numbers are entirely different things.*

When we discussed this problem with theoretical physicists they said they were not interested. When we contacted mathematicians, they replied that they knew the problem but for "rather well understood reasons" they do not want to get involved.

Anyhow, as we will study later, the concept of infinitesimal used in fluid dynamics and (quantum) field theory causes some difficulty as it is nothing but a naive concept for this most intrinsic and delicate concept of mathematics. Basically their concept of infinitesimals is as naive as Leibniz's. In QED, Feynman correctly pointed out that the divergence problem occurs because of this. Unfortunately, Robinson's "correct" infinitesimals does not relate to the needs of physics either. Yet there seems to no communication between mathematical physicists and pure mathematicians.

2 Energy?

2.1 Energy, kinetic energy and the conservation of energy

The kinetic energy of mv is defined as the work needed to accelerate from $m0$ to mv. Contrary to the common belief, the work discussed here depends upon the way we accelerate. When we accelerate with constant acceleration, the work needed is $mv^2/2$. But in general it is not true. So, under the standard definition that *energy is the potential to do work*, the assumption that the kinetic energy of mv is $mv^2/2$ is in violation of the conservation of energy.

Remark 4 *This definition of energy clearly indicates that this concept is not a physical reality but a modality.*

When the acceleration is constant, certainly the work needed is as above. But this is not the case if the acceleration is not constant except a few other cases. Let us demonstrate this point with an example. Assume $a(t) = t$ instead of constant. Then for the initial condition $v_0 = x_0 = 0$, we have the equation of the motion as

$$v(t) = \int tdt = \frac{1}{2}t^2, \quad x(t) = \int \frac{1}{2}t^2 dt = \frac{1}{6}t^3.$$

So, we have

$$W = \int (mt)x(t)dt = \int \frac{1}{6}t^3 mtdt = \frac{m}{24}t^4.$$

However,

$$\frac{m}{24}t^4 \neq \frac{mv^2}{2} = \frac{m}{2}(\frac{1}{2}t)^2 = \frac{m}{8}t^2$$

as an identity of t. *So, this issue invalidates a most important law of physics, which is the Conservation of Energy Law.*

This problem of mismatch between energy and work yet has another probably more important consequence. Work can cancel each other as work can be negative. However energy can not cancel as energy is always positive. Consider that with work W_1, we accelerate from position A with momentum $m0$ to position B with momentum mv and from here with work $-W_1$ symmetrical we decelerate from mv to the position C with momentum $m0$. The work done is 0.

However the energy needed to do this work is $2E$ where E is the energy needed to do the move m from A to B. This means that energy $2E$ is used to do no work. So, obviously the conservation of energy is false.

One of the main difference between the legitimate concept of momentum and the troubled concept of energy is that energy as defined is a "modality", what modal logicians call "possibility". It is an acknowledged fact that combining predicate reasoning and modal reasoning could lead to inconsistency. So, this might well be a culprit of the inconsistencies theoretical physics is facing in so many places. Certainly it is not quite plausible that modal concept such as energy can be the most fundamental law of physics which is supposed to be the precise most physical science.

2.2 Relativistic kinetic energy?

In classical theory, collision problem steps out of dynamics as there is no good way to describe the moment of collision. So, they describe only pre-collision process and post-collision process and compare them to describe the collision. To avoid the difficulty of dealing with multi-body problem they also assume that there is no material mutually attracting or repelling force among the multi-bodies. However, even under this restriction, we still have some serious issues to be dealt with.

Assume m and M are to collide in the absolute frame with relative speed v,which is the difference between the absolute speed of m and that of M in the sense of Newton dynamics where $M >> m$. As usual, we assume that there is no acceleration on m and M so that the conservation of momentum holds. Then energy wisely from m's perspective, M is to hit m with kinetic energy $Mv^2/2$ and m is to hit M with kinetic energy $mv^2/2$. Certainly, there is a problem with all of this. This is a very simple explanation for why relativistic theory in general is not acceptable.

After all, there are some good reasons why Newton rejected both energy and relative reference frames. This also constitute a warning to contemporary theoretical physics not to mix momentum with the invalid energy as Einstein did through his momentum-energy equation as we discussed in our first paper. [KPW-1]

3 Wave particle duality of de Broglie and Schrödinger's equation

De Broglie made a huge impact on the issue of *wave particle duality*, which is a special case of more general *field-particle duality* of quantum field theory.

3.1 De Broglie's relativistic wave particle duality

De Broglie obtained the following relativistic transformation for a plane wave which is invariant under the Lorentz transformation (we call it a "relativistic wave"):

$$k'_x = \frac{1}{\sqrt{1-(v/c)^2}}\left(k_x - v\frac{\omega}{c^2}\right), \quad k'_y = k_y, \quad k'_z = k_z, \quad \omega' = \frac{(\omega - \nu k_x)}{\sqrt{1-(v/c)^2}}$$

where $\mathbf{k} = (k_x, k_y, k_z)$ is the wave vector and ω is the frequency. We denote the wave number $|\mathbf{k}|$ by k. So, $k = |\mathbf{k}|$. This restriction to "relativistic waves" is because otherwise the wave phase $\mathbf{k}\cdot\mathbf{r} - \omega t$ will not be invariant under the Lorentz transformation. Here v is the speed of the wave observed in the frame of the observer.

The condition of being "relativistic wave" is a strong one. So far, all we know is that a wave is relativistic if it is a solution of a wave equation which is invariant under the LT. As we have shown in the first presentation "Logical Analysis of Relativity Theory", contrary to the folklore most wave equations are not invariant under the LT. Semantically, this is not surprising at all as most waves have little to do with the speed of light. For the same reason, it is highly questionable to take Lorentz transformation as the most important transformation in Physics.

Using nothing but the *"syntactic analogy"* between this and the momentum-energy transformation of relativity dynamics of Einstein,

$$p'_x = \frac{1}{\sqrt{1-(v/c)^2}}\left(p_x - v\frac{E}{c^2}\right), \quad p'_y = p_y, \quad p'_z = p_z, \quad E' = \frac{(E - \nu p_x)}{\sqrt{1-(v/c)^2}}$$

where $\mathbf{p} = (p_x, p_y, p_z)$ is the momentum vector and E is the energy, de Broglie "proposed" the following association between a particle and a wave (called matter wave):

$$\mathbf{p} = \hbar\mathbf{k} \quad E = \hbar\omega$$

where $\hbar$ is a constant. We call this *"de Broglie (wave-particle duality) relation". This was how the general wave-particle duality of Quantum Mechanics was introduced.*

This was in analogy to the Einstein's wave particle duality between electromagnetic waves and photons

$$p = h/\lambda \qquad E = h\nu = pc$$

which turned out to be invalid at the pain of mathematical contradiction.

Lorentz transformation is defined in terms of the constant c, which is the speed of electromagnetic waves in vacuum. So, there is no ontological reason to think that this transformation will conserve wave functions which are not electromagnetic wave equations of Maxwell. Most waves have nothing to do with the speed of light.

Remark 5 *Something similar already happened. We showed in the other paper for this conference that Lorentz transformation fails to preserve the second law of dynamics for example.*

In the foregoing, we discussed the wave particle duality of light wave and photon as per Einstein and that of matter wave and relativistic particles as per de Broglie. Despite this promising "analogy", there is a fundamental difference between these two duals. *Light-photon duality of Einstein does not lead to the so called "uncertainty principle"*. Indeed, if position-momentum uncertainty holds for photon-light, we will never observe photon (light) as the speed of photon is constant c. Contrary, de Broglie's particle-wave duality as embedded in von Neumann's formalism (more essentially Schrödinger's formalism as von Neumann's formalism is a variation of Schrödinger's) leads to uncertainty principle. *So, de Broglie's wave-particle duality is not quite a generalization of Einstein's em wave photon duality.* Of course, this is putting the inconsistency of particle wave duality, de Broglie's version or Einstein's photon-em wave version, aside.

3.2 Tension between de Broglie and Einstein: scientific opportunism

There is some serious tension between Einstein and de Broglie which is not often discussed. The wave-particle duality as above leads to the following relativistic invariance relation $(\omega')^2/c^2 - k' = (\omega)^2/c^2 - k$. De Broglie's hypothesis was that associated with a particle with speed v, was a wave of phase speed w. The group speed $v_g = d\omega/dk$ under the above mentioned relativistic invariance relation leads to $2\omega/c^2(d\omega/dk) - 2k = 0$. From this we get $v_g = d\omega/dk = c^2(k/\omega)$. As the phase speed is $w = \omega/k$, we have

$$v_g w = c^2$$

This means that either the group speed or phase speed must be larger than c.

For de Broglie, this did not register as a contradiction. Under the duality between wave and particle which he "established" as above it must be that a particle can move faster than c. Unless otherwise the last equation above is false. De Broglie's response to this criticism was that *"as energy travels with the group speed, everything is OK."*

Remark 6 *Clearly, this contradiction came from the "de Broglie (wave-particle duality) relation". In physics, wrong theoretical results could readily be safely "mended" by the "empirical consideration" like this. This "opportunism" is an abuse of both mathematics and empiricism. Empiricism should never be used to fill the hole theoreticians created. Another example is the way how theoretical physicists introduced spin of electrons. (see 3.3.4 Empirically most verified theory?) Theory should stands on its own feet and face the judgement of empiricism. Is this not the way how things should work in empirical sciences? There is a little more to physics than just building steam locomotive engine.*

3.3 Schrödinger's quantization of Hamiltonian dynamics

3.3.1 Schrödinger's wave equation

Schrödinger used Hamilton's energy dynamics for the particle theory and applied de Broglie's pilot wave theory to produce a wave-particle duality which looks after the energy issue of de Broglie's relation.

All wave propagated along the x-axis obey the following wave equation

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{\omega^2}\frac{\partial^2 \Psi}{\partial t^2}$$

where $\Psi(x,t)$ is the wave function and ω is the wave speed.

Here, we consider the wave function Ψ whose square yields the probability of locating a particle at any point in the space. We consider only systems whose total energy E is constant and whose particle move along the x-axis and are bound in space. Then the frequency associated, via "de Broglie relation" which is totally hypothetical and relativistic, with the bound particle is also constant, and we can take the wave function $\Psi(x,t)$ to be $\Psi(x,t) = \psi(x)f(t)$. As the frequency is assumed to be precisely defined, $f(t) = \cos 2\pi\nu t$. So, we have

$$\frac{\partial^2 \psi}{\partial x^2} = -\left(\frac{2\pi}{\lambda}\right)^2 \psi = -\left(\frac{p}{\hbar}\right)^2 \psi$$

where the wave length is $\lambda = \omega/\nu$ and the momentum of the particle is $p = h/\lambda$.

We take the particle of mass m to be interacting with surroundings through a potential-energy function $V(x)$. The total energy of the system is given by

$$E = E_k + V = p^2/2m + V$$

where E_k is the kinetic energy of the particle. Then we have $p^2 = 2m(E - V)$. This leads us to

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi = E = i\hbar\frac{\partial \psi}{\partial t}.$$

This equation, is called the *Schrödinger wave equation*.

This equation is a "relativistic" wave equation for Hamilton's particle equation. In reality, it is not invariant under the Lorentz transformation. This does not mean that Quantum Mechanics is a non-relativistic theory. The derivation of Schrödinger wave equation involved de Broglie relation which is relativistic. Here we have ended up with a situation where we have an important equation which is "half relativistic and half classical". Schrödinger knew this problem and he tried to make his wave equation relativistic without success.

3.3.2 Schrödinger's Uncertainty Principle

Schrödinger tried to reverse the process of converting particle equation to wave equation to complete the desired wave-particle duality. Here he encountered a difficulty. One of the most fundamental issues in measuring waves is that there is an inherent uncertainty (ambiguity) in measuring the waves is that it must involve the measurement of counting the number of cress passing at one fixed point (observing point) in the path of the wave. In general, we have a difficulty in deciding what to do with the situation where the point is in between two cresses passing at the moment of measurement. This problem always manifests when we try to measure the material waves. This problem is called the "fundamental uncertainty problem", in symbols

$$\Delta\nu\Delta t \simeq 1.$$

Since $E = h\nu$ the uncertainty above brings in the following uncertainty: $\Delta E\Delta t \simeq h$.Similarly upon the measuring of momentum/location, we have the following uncertainty: $\Delta p\Delta x \simeq h/2$.

Unfortunately, this problem dose not disappear in a more advanced (mathematically) formulation of Quantum Mechanics due to von Neumann. This is because his Hilbert space formulation still is based upon Schrödinger's wave equation which is based upon the wave-particle duality of de Broglie. As we will discuss later, *adding probabilistic aspects, as per Max Born, to Schrödinger and Heisenberg, von Neumann derived the same uncertainty result probabilistically without using the uncertainty of wave measurement. But they excluded the energy/time uncertainty of Schrödinger due to the commuting operator restriction. Physically, it mat appear that Schrödinger's theory is richer because of this. But operators are directly linked to observations and from this point of view, it appears that exclusion of energy/time uncertainty is important. The problem with energy/time is that neither of them are physical entities. Energy is modality and time transcends mathematics. So, certainly neither of them are really measurable. This failure of the commutativity is a manifestation of this deeper philosophy.*

3.3.3 Uncertainty principle v.s. trajectories

The ultimate destiny, as presented by Ballentine, of this attempt to cross the line between continuum and discrete is the *"empirical refutation" of the theory of quantum mechanics*. Uncertainty of quantum mechanics predicts that once we localize a particle, the momentum becomes impossible to be predicted, resulting in the impossibility of observing trajectories of particles. In all particle collider experiments, particles are producing trajectories and physicists use these trajectories to study particle world. *Just for information, the resolution of the localization we are discussing is at the level of water molecule.* In this way, quantum mechanics took the empiricism away from physics. But in many sense, this seems to make more sense than otherwise. Though it has been claimed that quantum theory is the most empirically verified theory in the history of physics, it appears that this theory also is the most empirically refuted theory in the history of physics. So far, the most "convincing?" response from the quantum physicists is that it is not the fault of QM but it is the fault of Copenhagen interpretation. This is not the issue of interpretation!

Regarding this problem, Kuhn pointed out that probabilistic prediction does not constitute empirical prediction as the relative frequency converges only at the limit.

3.3.4 Empirically most verified theory?

(1) In contrast to what we discussed above, Quantum Electrodynamics is known as "the most empirically verified theory" of physics in history. *This in turn supports the "ultimate empirical validity" of Special Theory of Relativity which is the most logically refuted theory.* What is most astonishing is that all of these claimed experimental verifications of this theory came from trajectories produced in the particle detection chamber. Curiously, the UP of QM asserts that there must be no trajectories of particles observed. As usual, empiricism is used intensively to support the theory when it works and when it stands against the theory, they ignore it. There is a commoner's word for it, "opportunism". These things do not happen in pure mathematics, just for information.

We will also question the role of relativity theory in interpreting the experimental results coming from the trajectories produced in particle detection apparatuses. The trajectories are caused by the (charged) particles colliding with the gas molecules inside the chambers. This means that special theory of relativity does not apply in the chamber. There must be no acceleration in STR. Despite this, all theoretical analysis of the trajectories are based upon relativistic formulas. To make the matter even more confusing, there seems to be no frame moving in this situation. Our frame is the frame of the particle detecting chamber and particles are moving in it. So, why we need relativistic theory here.

What is becoming rather clear is that in physics, fancy theories (with mathematical and conceptual incoherence) are developed and experimentalists apply these fancy formulas to their experiments rather mechanically without due analysis of the experiments to determine if the formulas are applicable to their situation.

Above all, it may be worth while to figure out why predictions of this theory are "experimentally verified" with record high success rate? It is a very interesting question. To investigate this issue, it may be helpful to pay attention to what Bertrand Russell said on empiricism in a general setting. He said,

1. Empirical verification of a theory is a vicious circle as experiment to verify the theory uses the theory to be verified.

2. Empirical refutation of a theory is a contradiction as the experiment to refute the theory uses the theory to be refuted.

It was unfortunate that this did not attract attention from physics community. Physics's founding philosophy of "anti-logic" is sticking its head.

(2) Going back to the original question: there is an "interesting answer". In particle physics experiment, as particles are way too small to be picked up

and examined as we do in classical physics, we rely upon the trajectories particles leave and apply formulas of Quantum Electrodynamics to measure. *So, measurements are done as calculations. Then naturally the formulas used to calculate are vacuously verified.* It is a vicious circle.

Example 7 *Here is an example which reveals this problem: we will examine a typical argument of particle physics in which they experimentally find an unknown particle mass in terms of known masses and measured energies and momenta. The experimental data is the trajectories which appeared in a bubble chamber violating the Uncertainty Principle which theoretically refutes any chance of observing trajectories as we have been discussing through out. In this experiment a beam of very high energy negatively charged k^{-1} masons enters the bubble chamber, The following two possible particle reactions takes place*

$$k^- + p^+ \to \Lambda^0 + \pi^0 \qquad K^- + n \to \Lambda^0 + \pi^-$$

where p^+, n are proton and neutron respectively and Λ^0, π^0 and π^- are uncharged hyperon uncharged pion and negative pion. As Λ^0, π^0 are neutral electronically, we do not observe the tracks of them. However, the following particle interaction follows: As the resultants of this reaction are both charged, we can observe the trajectories of them (p^+ and π^+).

$$\Lambda^0 \to p^+ + \pi^+.$$

At a point A, a k^- track disappears due to the equation for k^-. A Λ^0 is produced which travels to point B where it decays into p^+ and π^+. The direction of π^+ relative to AB is θ_π and that of p^+ is θ_p. We will apply relativistic laws of conservation of energy and momentum to this experimental data of the decay of Λ^0 at B for the measuring of the rest mass $m_{0\Lambda}$ of Λ^0. We assume that the speed of Λ^0 is v_Λ and its energy is

$$E_\Lambda = m_{0\Lambda}/\sqrt{1 - v_\Lambda^2/c^2}.$$

Then we have

$$E_\Lambda = m_{0\Lambda}/\sqrt{1 - v_\Lambda^2/c^2} = E_\pi + E_p \qquad (1)$$

where E_π and E_p are relativistic energy of pion and proton. By the conservation of relativistic momentum, we have

$$\begin{aligned} p_\Lambda &= p_x \cos\theta_\pi + p_p \cos\theta_p \qquad (2) \\ 0 &= p_x \sin\theta_\pi - p_p \sin\theta_p \qquad (3). \end{aligned}$$

Dividing (2) by (1) and $p = mv = E_v/c^2$, we have

$$v_\Lambda/c = p_\Lambda c/E_\Lambda = (p_x c\cos\theta_x + p_p c\cos\theta_p)/E_\pi + E_p.$$

All of this leads to

$$m_{0\Lambda}c^2 = (E_\pi + E_p)\sqrt{1 - v_\Lambda^2/c^2}.$$

As

$$p = \sqrt{E^2 - m_0^2c^4}/c = \sqrt{T^2 + 2m_0c^2T}/c$$

where T is the kinetic energy, for the experiment with $T_p = 44MeV$ and $T_\pi = 60MeV$, we have $p_p = 291/cMeV$. The rest energy of π^- is $139.6Mev$, which leads to $p_\pi = 143/cMeV$.

Moreover, many other setting of the experiment leads to quite close calculated results. From this particle physics researchers claim that *the Relativity Theory is experimentally verified.* What is happening here is what logicians call a vicious circle. As we explained above, *If we interpret some experiments using calculation (formula) of "a" theory, naturally "such theory" is verified by the "experiments" we do.* This is a near fatal logical issues for empiricism. *This is why relativity theory has been experimentally verified through QED most often in the history of physics despite that the theory is inconsistent.* The problem with this is that they did not measure the rest mass $m_{0\Lambda}$ of Λ^0 directly by measurement. This is a clear case where *traditional macroscopic empiricism does not connect to modern microscopic theories.*

(3) Here also is a problem which goes in *opposite direction.* The problem of spin in electron was introduced because of the experiment. Electron in magnetic field left trajectory (which are not supposed to be there), which was theoretically unexpected. They discovered that if we assume electrons spin then this phenomena could be explicated. So, the spin is added to the theory. Theory was "supplemented" by the experiment in this case. It is this kind of incoherent opportunism which turned off so many serious thinkers. In particle physics this became a routine exercise. *When ever they found some new phenomena the theory could not explain, it was added as a "new axiom" for the theory. Then they claim that their updated theory is empirically verified.* Nobody cared the maintaining of the consistency of such theory as nobody in theoretical physics understood consistency. For physics, logic has been arch enemy from the religious revolution time. Logic was directly associated with Vatican cosmology. Now we have the "standard model" thus obtained.

Remark 8 *One may say that theory is ignored for "convenience" too. A good example is that the em field theory predicts the speed of light is dependent upon the frequency in the presence of the electric current. Without current there is no emission of em waves as em waves are emitted by accelerating electrons. So, it must be that the speed of light is relative to the frequency of light. This is consistent with the classical optics theory. As this is inconvenient for the belief system of Einsteinian relativity theory, this theory is ignored. So, it may be interesting to measure the speed of light through plasma if we have technology to do so.*

3.3.5 Is uncertainty principle really valid?

After all, the UP as per Schrödinger came from the wave particle duality of de Broglie which was motivated by Einstein's em wave v.s. photon duality. We have shown the illegitimacy of Einstein's result and de Broglie's work. We also showed that the way de Broglie presented the duality is invalid. This makes us seriously question the validity of Schrödinger's UP. The fact as stated above that in all particle experiments which produces trajectories UP is experimentally refuted appears to be ultimate empirical refutation of at least UP if not the entire QM. We wonder when and why the most basic principle governing physics that experiment refutes theory, not the other way around, was abolished.

More articulately, it may not be the fault of just UP itself. It appears that all of this mystery is pointing finger at the way how quantum mechanics which produced UP was developed. After all, UP was developed upon inconsistent theory of relativity. It is a logical trivia that an inconsistent theory can prove anything and this is why anybody with any rational mind will throw away inconsistent theory.

One may say that there is such thing as the "empirical equivalence" of Schrödinger's formalism and Heisenberg's formalism. Then, regardless of the theoretical validity issue of Schrödinger and Heisenberg, it appears that von Neumann was correct in combining Schrödinger and Heisenberg in his new formalism for QM. "If so", all troubles we encountered in the "empirical results" of Schrödinger such as the UP must be shared by Heisenberg's formalism. So, they share the same head ache on all debatable empirical results. Putting aside this issue, a major question here regarding von Neumann is the validity of probability issue. Regarding this, Kuhn made an incisive analysis and refuted probability aspect of QM. He said, as the relative frequency converges only at the limit, we can not even refute QM's prediction empirically, never mind the verification. Ignored and disappointed, Kuhn left physics.

Logically speaking, it is interesting that on the one hand, Schrödinger's formalism can not be made relativistic and this is not the case with Heisenberg's formalism. Are they really equivalent? *This question has a massive impact on the achievement of Dirac who assumed the empirical equivalence of Heisenberg and Schrödinger to develop QED.* Upon this "claimed equivalence" the highly questionable exercise of Gordon-Klein became standard in quantum field theory and they mixed up these two formalisms. *The unsolved difficulty of making Schrödinger equation relativistic was "resolved!?" by Gordon-Klein by replacing energy variable and momentum variable in the "invalid" energy-momentum relation of Einstein with energy operator and momentum operator of Heisenberg.* Quite interesting method of putting two mutually contradicting theories together indeed. *If this works, why we need wave equations to begin with? Just start with classical equation and replace variables with non-commuting operators.* Then Quantum mechanics becomes triviality, does it not?

Remark 9 *They may well try to defend their move saying that due to the claimed empirical equivalence between Hisenberg-Jordan's formalism and Schrödinger's formalism, they can first derive the energy-momentum relation of Einstein and replace energy variable and momentum variable with energy operator and momentum operator as Gordon-Klein did. Unfortunately, regardless of if the energy-momentum relation of Einstein is a theorem of Heisenberg-Jordan, this relation* *is invalid as it came from the ill fated* $e = mc^2$*. Moreover, it makes no logical sense at all either. Energy is not physical quantity. It is modality and momentum is physical quantity and there is no point in having an equation which relates these two entirely unrelated quantities. Just a little bit of elementary logical training would have diverted theoretical physics from this devastating and embarrassing error.*

In the 20th century, study of mathematics and logic made astounding progress and the "reasoning" theoretical physicists use almost everywhere in their theory is not up to date anymore. *One cannot develop 21st century theoretical physics with 19th century mathematics and logic.*

4 Heisenberg's quantum mechanics

As a reaction to Bohr's Quantum Mechanics of Correspondence Principle, Heisenberg [Heisenberg] came up with desirable infinite matrix representation of physical quantity and coordinate using Fourier components with which he converted Hamiltonian equation of motion into infinite matrix version of canonical equation of motion:

$$dp/dt = [(2\pi i)/h](pH - Hp), \qquad dx/dt = [-2\pi i/h](xH - Hx).$$

The first equation describes the quantum systems as Hamiltonian operator. The second equation describes the canonical equation of motion. With them we calculate the absolute values of the amplitude of momentum p and location x. Also we calculate the frequency of the phase of p and x. There is no mention of energy and time as observables in this theory.

Remark 10 *This is consistent with the fact that energy is not a legitimate concept for two reasons as we already discussed: First energy is not a physical concept. It is a modality. Second unlike location, as we have discussed many times time has a special feature which flows forward on its own which can not be described by any mathematical or physical means. This means that time and energy are metaphysical concept and in dynamics real physical quantities are location and momentum. Unfortunately these issues are irrelevant details to be ignored for the main stream theoretical physics community. However, we must pay attention to the fact that the Hamiltonians are energy operators. The ghost of modality can not be shaken off easily. It is "tragic" reality that the theory of physics must depends upon this type of modality which has nothing to do with physical reality. However, time which is measurable is not modality and it is more fundamental and mysterious than energy. The reality we face is that without time there is no physics at all. As Feynman pointed out, modality like the force field could be removed from physics and he was correct.*

Schrödinger allegedly showed "empirical equivalence" of his Quantum Mechanics and Schrödinger's Quantum Mechanics. Physicists argue that, these two major theories one coming from the Hamilton's classical equation plus Bohr's correspondence principle and the other coming from Hamiltonian classical equation plus de Broglie's relation are *empirically equivalent*, "airs" that some thing at least "empirically intrinsic" is achieved.

Remark 11 *Original Heisenberg's theory is relativity free while Schröginger's theory is relativistic as it is depending upon the relativistic theory of de Broglie. This makes us doubt the claimed empirical equivalence of Heisenberg and Schrödinger. Considering the reality that for physicists mathematics and logic are just languages, it is hard to believe the issue here were understood as well as they should have been.*

Notwithstanding, as von Neumann pointed out, Heisenberg's Quantum Mechanics and Schrödinger's Quantum Mechanics are not quite equivalent at rather obvious level. For Heisenberg, energy is not measurable and it is measurable for Schrödinger. So, the "claimed equivalence" is limited to just the measurement of momentum and location.

Contrary, mathematicians do not understand theoretical physics or what theoretical physicists are up to and just keep distance gossiping about "how ignorant theoretical physicists are". They do not want to understand what is happening in physics and this is yet another side of the problem we are facing. *It in fact appears that the empirical equivalence of Heisenberg's formalism and Schrödinger's formalism breaks down at the Uncertainty Principle.* Heisenberg's UP comes from the idea of Göte who said that when we make measurement we impact the physical system and change it. He was rightly pointing out the deadly reality of empiricism. Schrödinger was talking about the intrinsic difficulty in measuring the waves coming towards us.

So, *Göte-Heisenberg uncertainty has little to do with waves. On the contrary, Schrödinger's UP seems to have little to do with the impact we give to the wave when we measure it.* Ontologically, the claimed empirical equivalence of Heisenberg's formalism and Schrödinger's wave mechanics have little do to with each other. *The current state of the so called wave-particle duality breaks down here.*

This issue is closely bound up with the issue of Dirac's quantization which apparently tries to quantize mathematical continuum. Dirac certainly knew little about set theory and the foundation of mathematical analysis which are closely bound up together to explicate the mathematical continuum.

Physicists are arrogant very directly. Mathematicians are real snobs who just laugh at theoretical physicists without understanding the real difficulty theoretical physicists are facing. We have a serious difficulty sitting right in the middle of mathematics and physics and nobody has courage to accept the problem and try to solve it.

5 Von Neumann's Formalism

As a "consequence of de Broglie relation", a particle with energy E has a wave function $\Psi(x,t) = Ae^{i(px-Et)/\hbar}$. It then follows that $i\hbar\,\partial\Psi/\partial t = E\Psi$. This together with the Schrödinger equation above will give us

$$(\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - V)\Psi = E\Psi = i\hbar\frac{\partial}{\partial t}\Psi.$$

So, we can consider $E = \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - V$ as a representation of all energy issues of this particle. It is called Hamiltonian. We denote it with H. So, Schrödinger equation can be written as

$$i\hbar\frac{\partial}{\partial t}\Psi = H\Psi.$$

where the Hamiltonian can represent more general energy situation of the particle.

Heisenberg and Jordan presented an alternative formulation of early Quantum Mechanics using matrix and linear operators where operators are used to represent measurements. Though they were "shown?" to be "empirically equivalent", these two groups did not agree with each other. It was von Neumann who unified Heisenberg-Jordan theory and Schrödinger theory and added Born's view of probabilistic multiplicity in measurement. His new idea was to represent measurement as Hermician operators and use the eigenvalue analysis applied in probability theory to yield the probabilistic distribution of the measured values as eigenvalues.

Axiom 1: In the absence of external influence, the quantum state $\vec{\psi}$ changes smoothly in time t according to the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\vec{\psi} = \hat{H}\vec{\psi}.$$

where $\hat{H}$ is the Hamiltonian operator over the complex Hilbert space.

Axiom 2: An observable is a self-adjoint linear operator over the complex Hilbert space.

Axiom 3: [Projection Postulate] When we apply an observable $\hat{A}$ to a quantum state $\vec{\psi}$ (we call such application an observation $(\hat{A}, \hat{\psi})$), then the quantum state jumps into one of its eigenvector and one of the eigenvalues of the eigenstate is observed.

Axiom 4: The following represents the probability of one of the eigenvalue a of $\hat{A}$ observed:

$$PR((\hat{A}, \psi) \approx a) = \left\langle \vec{\psi}, \hat{P}\vec{\psi} \right\rangle$$

where $\hat{P}\vec{\psi}$ is an eigenstate of A with eigenvalue a.

Remark 12 *As is clear in this formulation, the standard Quantum Mechanics due to von Neumann yields the prediction on the "probabilistic distribution" of possible measurement values for each quantity to be measured. The uncertainty* *problem first appeared in Schrödinger's formalism of Quantum Mechanics as a consequence of making measurement to waves. It was a well known issue of wave mechanics. This problem in von Neumann's formalism is reduced to a problem of purely mathematical nature. Axiom 4 above is a standard convention in Statistics to combine probability theory and linear algebra.*

One of the major urge for developing Quantum Mechanics was the failure of old Rutherford model to explicate the discrepancy between the empirical reality and theoretical prediction of classical em theory that the orbiting electron in the hydrogen atom must fall into the positron as the continuous loss of kinetic energy by emitting em wave. The Hamiltonian energy equation for such system of electron orbiting around the positron does contradicts this empirical facts that electron never falls into the positron. It just makes little sense to claim that by considering relativistic pilot wave to the orbiting electron as Schrödinger did we can resolve this problem at all. As discussed before, Hamiltonian dynamics and pilot wave theory are mutually inconsistent as the former is classical and the latter is relativistic. It is highly irregular to think that combining two mutually inconsistent theory will yield a consistent theory.

To make the matter even worse, de Broglie relation comes from Special Theory of Relativity which is inconsistent. This clearly establishes that the von Neumann's Quantum Mechanics is inconsistent.

A criticism as serious as the consistency issue if not more is the connection of this product of a mathematician, von Neumann, to Physics. Projection Postulate has no ontological meaning. So, just like Minkowski's 4D spacetime relativity theory, this theory seems to have little relevance to Physics. This problem was pointed by philosophers and was systematically ignored by the main stream physics community.

From the von Neumann's formalism we can also derive the following "generalized uncertainty principle".

Generalized Uncertainty Principle: Let x and p be observables (measurements) such that $[x, p] = xp - px = i\hbar$. Then

$$\Delta x \Delta p \geq \hbar/2$$

where dispersions of x and p respectively.

This important result can readily be obtained from the von Neumann's axioms and Schwarz inequality of Hilbert space.

6 A passage to "fully grown" quantum electrodynamics

Putting aside the fatal fact that relativity theory is inconsistent, in Quantum Mechanics, Schrödinger faced a difficulty in trying to make his wave equation relativistic even thought the equation was obtained using (both classical dynamics and) relativistic de Broglie wave particle duality.

6.1 Gordon-Klein's revolution

Gordon-Klein, instead, made a most fundamental relativistic equation of energy momentum relation $E = \sqrt{(cp)^2 + m_0^2 v^4}$, (which is a consequence of the ill fated $E = mc^2$) quantized yielding a "quantization of relativistic dynamics". They essentially replaced all physical variables in the energy-momentum equation with corresponding self-adjoint operators of Quantum Mechanics as in the quantization of Hamiltonian equation. So, quantization was reduced to the routine of *replacing classical quantities in the classical equations with "corresponding" self-adjoint operators.* A leap forward indeed.

Unfortunately, Gordon-Klein's "quantization" of invalid relativistic energy-momentum equation of Einstein does not offer a convincing solution to this fundamental problem that Schrödinger's wave equation is not relativistic. Replacing relativistic energy variable and relativistic momentum variable with energy operator and momentum operator in the faulty relativistic energy-momentum relation is not what we should call quantization. If Gordon-Klein's work was quantization, there was no need for Schrödinger's equation to begin with.

More over, the Gordon-Klein equation does not conserve probability, which is one major requirement imposed by the usual interpretation of quantum mechanics. Quantum mechanics interprets the square of the modulus of a wave function's amplitudes as probabilities. For that reason, Schrödinger's equation was made to make sure that the coefficients of wave functions were normalized at every point in time. This unfortunately is not the case for the Gordon-Klein equation. Needless to say, it cannot thus be seen as a valid replacement for a relativistic version of Schrödinger's equation. In order to conserve probability, a time evolution equation needs to satisfy the following condition with regards to a wave function $\int \left|\psi(x,t)^2\right| dx = 1$.Furthermore, as the conservation must hold at any point in time, it has to be independent of time evolution. This is to say that the Gordon-Klein equation must satisfy the following equation as well

$$\frac{\partial}{\partial t} \int \left|\psi(x,t)^2\right| dx = 0.$$

Now consider the Gordon-Klein equation

$$\frac{1}{c^2}\frac{\hbar^2 \partial^2}{\partial t^2}\psi(x,t) = (\hbar^2 \nabla^2 - m^2 c^2)\psi(x,t).$$

Since it involves the second derivative with respect to time, it is clear that the first derivative term in the probability conservation expression will in general not disappear. Hence, the expression will not produce the required value 0 and therefore the Gordon-Klein equation clearly does not describe the probability wave that the Schrödinger equation describes.

Notwithstanding, the most important issue is that relativity theory as per Einstein is inconsistent and there is no point in trying to make classical theories relativistic. Classical theories such as Electromagnetism theory have their own problems. Relativity theory is a wrong answer to the problem of classical electromagnetic theory.

6.2 Dirac's anti-electron

Upon this seriously flowed Gordon-Klein equation, Dirac's "relativistic quantum equation" of free electron was obtained. This lead to the famous "theoretical discovery" of anti-electron (positron). This was the start of "the most successful theory" in the history of physics, namely the Quantum Electrodynamics. From the start, Quantum Electrodynamics was off the track. Unfortunately all of this means nothing for the King of Science as mathematics is just a language for them. They have more important issues to attend.

Upon the "theoretical discovery" of anti-electron by Dirac as above, Yukawa pushed the boundary of Quantum Electrodynamics and explicated the mystery of why neutron and positron form a strong bond to form the nuclear of atoms. He explained this using a hypothetical particle called meson which positron and neutron pulls in towards themselves. The action reaction law of Newton then makes these two particles bound together. A most fundamental flow of this theory of Yukawa is that on the one hand, as a field theory, his theory violates the Action Reaction Law and yet his theory uses this law to bound neutron and positron. The tragedy is that Yukawa just like everybody else did not know or did not want to know that field theory violates the Action Reaction Law.

In the end, what is untenable here is to resolve the conflict between the field theory which violates the Action Reaction Law (or equivalently the Conservation of Momentum Law) and particle theory which operates upon this law. Does Born's probability interpretation of Schrödinger's wave equation help to smooth out this problem? Well probability seems to have the effect of making annoying problems disappear behind the haze of probability. It certainly had a strong effect on quantum mechanists, as Kuhn complained.

There is one more very fundamental problem with the work of Gordon-Klein. Putting aside the invalidity of the energy-momentum relation of Einstein as discussed above, the issue of category difference between the concept of momentum and energy is not considered in this work. Momentum is a physical reality but energy is not. It is modality. It is not clear at all how modal operator of energy can be obtained from the reality operator of momentum. This after all is a problem which Gordon-Klein inherited from Einstein.

6.3 "Approximation" removes another contradiction: Hegelean dialectic works

As a related problem, a violation of momentum conservation appears often in Quantum Electrodynamics. For example, it appears in the so called pair generation $\gamma \to e^- + e^+$ where a photon of high energy (γ ray) vanishes producing electron and positron. Let v be the (initial) speed of positron and electron produced. Let m_0 be the rest mass of electron and positron. Then as the conservation of energy and momentum conservation, we have

$$hf = 2m_0c^2/\sqrt{1 - v^2/c^2}, \qquad hf/c = 2m_0v^2/\sqrt{1 - v^2/c^2}.$$

This leads to a contradiction $v = c$.

Particle physicists deal with this difficulty through "approximation". When a gamma ray passes through "near by" a huge mass, this mass absorbs very small amount of energy and absorbs most of momentum involved. In this way in approximate we can "ignore" the momentum conservation and the above mentioned contradiction is "resolved". They claim that this happens when the energy of the gamma ray is more than $2m_0c^2$. There is no explanation on how this absorption of energy and momentum occurs.

What is irregular from the scientific common sense is that *this contradiction came from the invalidity of the development of Special Theory of Relativity Dynamics. Yet physicists resolve this fatal logical/mathematical error using "approximation"*.

Remark 13 *The true absurdity of this argument is that the momentum conservation is used. However, usage of gamma factor clearly suggests that this argument uses Einstein's STR dynamics. Putting aside the inconsistency and so invalidity of STR dynamics aside, the STR dynamics relates relativistic energy e and relativistic momentum p as*

$$e = \sqrt{(pc)^2 + (m_0c^2)}$$

Notwithstanding, Dirac introduced a new paradigm which connects Schrödinger's waves back to particles again. He Fourier expanded wave functions of Schrödinger and obtained many other particles associated with the original particle which was quantized by Schrödinger. Despite some questions regarding the perturbation method involved and some other basic issues regarding Quantum Mechanics in general as discussed above, he enriched the entire field mechanics of the original particle as thus expanded system of particles. This dramatically enriched the world of quantum particles and particle wave duality. What is not quite clear is how the incompatible world of waves and particles are consistency integrated in this way.

One of the major concern is regarding the usage of perturbation method for converting Schrödinger's wave equations into particles (the so called "second quantization"). This method introduced particles moving backward in time for which there is no ontological interpretation. By definition, there is no such motion as time never evolves backward. Moreover, due to the fatal failure of relativity theory, Schrödinger's wave mechanics is also invalid. This is obtained from the assumption that the fundamental flow of time evolution moves only forward.

Remark 14 *Regarding time evolving backward, beyond opportunism, we expect some more explanation. If time evolves backward too, what about the speed of such evolution of time. Is it the same magnitude as the positive evolution? How does it connect to the TD in the realm of STR? Is it possible that forward time and backward time interfere with each other, even creating variety of tome evolutions including stopped time? After all the idea of backward time evolution was created from the ill fated relativity theory.*

7 Dirac's Quantum Electrodynamics

7.1 Harmonic oscillators

Following Gordon-Klein's short cut version of "quantization", Dirac quantized classical Hamiltonian H for harmonic oscillator, by replacing physical quantities in it with corresponding self-adjoint operators as

$$H_{osc} = p^2/2m + m\omega^2 q^2/2m \qquad (i)$$

where p and q are operators which satisfy the commutation $[p,q] = i\hbar$. Though the connection between this purely "formal" quantization and de Broglie's (or Schrödinger) quantization is not understood as well as it should be, this easier to work on quantization took over and became standard in contemporary quantum field theory

With p and q, we define the non-commuting operators

$$a = (m\omega p + ip)/\sqrt{2\hbar m\omega} \qquad a^+ = (m\omega p - ip)/\sqrt{2\hbar m\omega} \qquad [a, a^+] = 1..$$

Now we have

$$H_{osc} = (1/2)\hbar\omega(a^+a + aa^+) = \hbar\omega(a^+a + 1/2).$$

Define N as $N = a^+a$. It follows: (1) Eigenvalues of N are $n = 0, 1, 2, ...$ (2) If $|n\rangle$ is normalized then so are $|n \pm 1\rangle$ defined as

$$a|n\rangle = \sqrt{n}|n-1\rangle \quad a^+|n\rangle = \sqrt{n+1}|n+1\rangle.$$

If $|0\rangle$ is normalized, the normalized eigen vectors of N are $|n\rangle = ((a^+)^n/\sqrt{n!})|0\rangle \quad n = 0, 1, 2, ...$ These are also eigenvectors of H_{osc} with eigenvalues $E_n = \hbar\omega(n + 1/2) \quad n = 0, 1, 2, ...$The operators a and a^+ are called *annihilation operator* and *creation operator* respectively under the assumption that $|n\rangle$ represent a quantum state with n quanta.

In summary, the quantized Hamiltonian for harmonic oscillator can be expressed using creation operator a and annihilation operator a^+

$$H_{osc} = (1/2)\hbar\omega(a^+a + aa^+). \qquad (ii)$$

Anyhow, the intention of Dirac is to represent energy (harmonic oscillator) as a countably infinite collection of "photons" with associated annihilation operator and creation operator.

Unfortunately, all of this seem to have no clear ontological meaning at all. How in the world can we say that (ii) is the quantization of the Hamiltonian operator (i). To begin with we have no idea what quantization of an operator means in this context. Rather tedious but apparently fancy calculation and nothing more seems to be going on in this obscure argument. One of the major difficulty in following quantum mechanics is that nobody seems to know what they are talking about. All fancy but meaningless calculations leading into big proclamations which makes little sense in the end. We should be given clear meaning of "quantization of H_{osc}. If it was meant to be the "particle representation of the continuum operator H_{osc}," then we do have some serious questions to ask. How is it possible to represent an operator which acts upon continuum with countably many particles? It is clear that Dirac did not understand what infinity means at all. That arrogance and ignorance to claim in public that mathematics is just a language is sticking its ugly head here too.

It has been said that the harmonic oscillator as energy created the problem of the blackbody radiation which forced Planck-Einstein develop quantum mechanics. More recent study also showed that when we represent energy with monochromatic oscillator we have no blackbody problem and thus no need for quantum mechanics. We do not know if any follow up has been made on this result.

After all the issue of energy being modality, not being physical concept is sticking its ugly head again. So, Dirac's argument to create particles called photon from imaginary entity called harmonic oscillator is fundamentally troublesome. *He is trying to drive physical particles called photons out of modality called energy.*

We can not see the modal nature of energy in the equation

$$H_{osc} = p^2/2m + m\omega^2 q^2/2m$$

can we? Energy is modality as we know. It appears quantum physicists do not care as Dirac is a "genius". Like Einstein, he was given half-god status. The simple answer is that this equation came from Gordon-Klein's equation which was obtained from replacing Einstein's false equation of energy-momentum which was obtained from the *false equation* $e = mc^2$. Einstein from this $e = mc^2$ obtained the equation which expresses momentum in terms of the energy. Momentum is physical entity and energy is modality. In philosophy the difference between energy and momentum is called category difference.

Remark 15 *There is a very strong argument against the energy-momentum relation in wave mechanics too. In wave mechanics, no matter moves in the direction of the wave motion. It is just local vibration of the wave medium which moves creating the motion of waves. So, there is no concept of momentum in wave mechanics. Many wave mechanists complain that because of the Einstein's genius they had to introduce the momentum energy relation, which is invalid, for modern wave mechanics.*

Remark 16 *The reason why there is no reference to energy e in the H_{osc} is because Hamilton did not know that the kinetic energy of mv may not be $mv^2/2$ as we discussed in 2.1 Energy, kinetic energy and the conservation of energy.*

7.2 Quantization of electromagnetic field: Dirac's aether theory

Planck quantized energy of electromagnetic waves to deal with the blackbody radiation problem. Dirac went on to quantize the electromagnetic field which is supposed to be the medium for electromagnetic waves of Maxwell. This is called the *"second quantization"*. Unfortunately the same problem as quantizing energy in the space as discussed in the previous section immediately appeared in this attempt.

7.2.1 Scalar and vector potentials

According to the classical electromagnetism theory, there are a scalar potential ϕ and a vector potential $\mathbf{A}$ such that the electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ of Maxwell can be obtained as

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \boldsymbol{\nabla}\phi, \quad \mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}.$$

If there is no source of the field, we choose gauge (Coulomb gauge) such that $\phi = 0, \quad \boldsymbol{\nabla} \cdot \mathbf{A} = 0$. From these equations we can derive the Maxwell equation of electromagnetic fields. What is not quite clear is how the vector $\mathbf{J}$ of current comes into this argument. Current is a continuous flow of charges, is it not? Maxwell's equations includes current as in the generalized Ampere's law. Moreover, accelerating current is what creates em waves.

Anyhow we have the following "wave equation of vector potential".

$$\nabla^2 \mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} = 0. \qquad (I)$$

This means that vector potential $\mathbf{A}$ for charge free space is a wave. But as $\mathbf{E}$ and $\mathbf{B}$ are modality, $\mathbf{A}$ is not physical reality but modality. So, $\mathbf{A}$ is not a physical wave but *a modal wave.*

7.2.2 Quantization of electromagnetic field

Following Dirac, we make a *Fourier expansion of the electromagnetic field in a large cube of volume* $\Omega = L^3$ and take the Fourier coefficients as the field variables. We choose the boundary conditions to be periodic on the walls of the cube. This is

$$\mathbf{A}(L, y, z, t) = \mathbf{A}(0, y, z, t), \quad \mathbf{A}(x, L, z, t) = \mathbf{A}(x, 0, z, t), \quad \mathbf{A}(x, y, L, t) = \mathbf{A}(x, y, 0, t).$$

The Fourier series of $\mathbf{A}$ is given by

$$\mathbf{A}(\mathbf{x},t) = \sum_{\substack{\mathbf{k} \\ k_z>0}} \sum_{\sigma=1,2} \sqrt{2\pi\hbar c^2/\Omega\omega_k}\,\mathbf{u}_{\mathbf{k}\sigma}(a_{\mathbf{k}\sigma}(t)e^{i\mathbf{k}\cdot\mathbf{x}} + a_{\mathbf{k}\sigma}(t)e^{-i\mathbf{k}\cdot\mathbf{x}}) \qquad (II)$$

where $\mathbf{k}$ is a wave vector, $\omega_k = kc$ and $k = \langle \mathbf{k} \cdot \mathbf{k} \rangle$. The factor $\sqrt{2\pi\hbar c^2/\Omega\omega_k}$ is a normalization factor. $\mathbf{u}_{\mathbf{k}\sigma}$, $\sigma = 1, 2$ are two unit orthogonal vectors.

From (II) and (I), with

$$a_{\mathbf{k}\sigma}(0) = \ \mathit{if}\ k_z > 0\ \mathit{then}\ a^{(1)}_{\mathbf{k}\sigma}(0)\ \mathit{else}\ a^{(2)}_{-\mathbf{k}\sigma}(0),\ \text{where}\ a_{\mathbf{k}\sigma}(t)e^{i\mathbf{k}\cdot\mathbf{x}} = a_{\mathbf{k}\sigma}(0)e^{-i\mathbf{k}\cdot\mathbf{x}}$$

we have

$$\mathbf{A}(\mathbf{x},t) = \sum_{\mathbf{k},\boldsymbol{\sigma}} \sqrt{2\pi\hbar c^2/\Omega\omega_k}\,\mathbf{u}_{\mathbf{k}\sigma}[a_{\mathbf{k}\sigma}(t)e^{i\mathbf{k}\cdot\mathbf{x}} + a^{(1)^*}_{\mathbf{k}\sigma}(t)e^{-i\mathbf{k}\cdot\mathbf{x}})]$$

This leads to $da_{\mathbf{k}\sigma}(t)/dt = -i\omega_t a_{\mathbf{k}\sigma}$. This equation for all wave vector $\mathbf{k}$ and $\sigma = 1, 2$ can be considered as "*the equation of change of the electromagnetic field*".

Now the energy in the electromagnetic field (radiation Hamiltonian) is

$$H_{rad} = \frac{1}{8\pi}\int_\Omega d^3\mathbf{x}(E^2{+}B^2) = \int_\Omega d^3\mathbf{x}\left(\frac{1}{c^2}\left|\frac{\partial\mathbf{A}}{\partial t}\right|^2 + |\nabla\times\mathbf{A}|^2\right) = \frac{1}{2}\sum_{\mathbf{k},\sigma}\hbar\omega_k(a_{\mathbf{k}\sigma}a^*_{\mathbf{k}\sigma}{+}a^*_{\mathbf{k}\sigma}a_{\mathbf{k}\sigma}).$$

With this, we can consider the em field to be an infinite collection of harmonic oscillators. Now we have

$$H_{rad} = \sum_{\mathbf{k},\sigma}\hbar\omega_k(1/2 + a^*_{\mathbf{k}\sigma}a_{\mathbf{k}\sigma}) \qquad (III)$$

and $\hbar\omega_k/2$ is the *zero-point energy* of an oscillator. Then the zero-point energy of the radiation field $\sum_{\mathbf{k},\sigma}\hbar\omega_k/2$ is *infinite* as there are *infinitely* many oscillators.

As there are continuumly many wave vectors $\mathbf{k}$, *the number of photons in the empty space is uncountably many. This does not agree with physical ontology of particles.* Does this not means that photons are not particles?

This problem we have discussed is directly connected to the question of how many photons in the electromagnetic field. Photons are *"supposed to be"* physical particles. The problem here is that *if photons are to support mathematical continuum then photons can not be physical particles.* A collection of ontological particles can not form continuum. Planck-Einstein's quantization of light waves shares the same problem. *As there are continuumly many wave length for electromagnetic waves there must be uncountably many photons of Planck-Einstein which is not possible as long as photons are particles.*

Here, Dirac carried out the quantization of (local) electromagnetic field expressed by the vector potential $\mathbf{A}$. This is to produce *"quanta of electromagnetic fields"* as harmonic oscillators and the total energy of such electromagnetic field as the summation (integration to be precise) of the energy of such harmonic oscillators.

Unfortunately, this result suffers from serious "category errors". Electromagnetic field is not a physical reality. It is a counter factual *modality*. So, the produced quanta of harmonic oscillators must not be considered as physical particle. *They are just a mathematical representation of this metaphysical world of electromagnetic fields which does not exist in physical reality.* How can the concept of the spacial distribution of electric force per unit charge be a physical reality. In Dirac's eccentric world where symbolic calculation is the only truth, "objects" defined from counter factual modality through formal symbol pushing produces physical reality of "photons" whose connection to Planck-Einstein's photons is not presented at all.

Remark 17 *Putting all of this fantasy aside, a very basic question still stands still waiting for answer. Suppose we have an electric field created by just one electron. What is the total energy of the electric field? This simple question exposes the absurdity of the entire concept of electromagnetic field and Dirac's analysis upon it. This problem all comes from the failure to understand that em fields are not physical reality but modality. The contempt of philosophy inherent to the very culture of physics as anti-logicism is sticking its ugly head as the cause of this serious error in the development of the so called most successful theory of physics in history.*

Remark 18 *Moreover, there is yet another good reason to question Dirac's claim that photons are in essence the components wave functions which appears in the Fourier expansion of the electromagnetic field expressed by the vector potential* $\mathbf{A}$*. This means that Dirac's photon is an "infinitary object" and this does not go quite well with the assumption that photons are "the most basic elementary particle".*

Remark 19 *Recent study shows that electromagnetic fields should be represented as the system of monochromatic operators instead to prevent the problem of blackbody radiation. Though this by itself will not provide a solution to the problem of particle-wave duality which is a very deep mathematical and philosophical problem, it at least seems to "explain" the blackbody radiation. After all, choosing harmonic oscillator or monochromatic oscillator for photon's mathematical representation has no ontological reasoning. So, this is a good example of how quantum theory violates the empiricism. It is tragic that physicists who claim that mathematics is just a language abuse mathematics to deal with inconvenient empirical issues like this.*

The most fundamental issue is that *it is not the case that we empirically detected particles called photons and then we found a mathematical representation of them.* Photons a la Dirac are nothing but the bizarre creation of rather incoherent mathematical construction of Fourier expansions. Dirac decided that they are physical particles called "photons" without considering their connection to yet another kind of photons presented by Planck who refused to consider his photons particles.

7.2.3 Annihilation operator and creation operator

Again, following the steps of Gordon-Klein, Dirac further "*quantized*" the above presented quantization of the classical radiative field *by replacing the classical quantities* $a_{\mathbf{k}\sigma}$ *and* $a^*_{\mathbf{k}\sigma}$ *with self-adjoint operators.* We may write $a_\sigma(\mathbf{k})$ and $a^*_\sigma(\mathbf{k})$ for $a_{\mathbf{k}\sigma}$ and $a^*_{\mathbf{k}\sigma}$. We just consider $a_\sigma(\mathbf{k})$ and $a^*_\sigma(\mathbf{k})$ quantum operators.

We assume that the operators refereeing to different oscillators commute, that is $[a_\sigma(\mathbf{k}), a^*_\sigma(\mathbf{k}')] = \delta_{\mathbf{k},\mathbf{k}'}\delta_{\sigma\sigma'}$.

Remark 20 *The malpractice of mechanically replacing classical variables with self-adjoint operators and call it quantization, as started by Gordon-Klein is sticking its ugly head again. It was an ad patch up job by them to "resolve" the difficulty of Schrödinger's wave equation not relativistic despite that it used relativity theory in the form of de Broglie relation.*

The operator $N_\sigma(\mathbf{k}) = a^*_\sigma(\mathbf{k}), a_\sigma(\mathbf{k})$ then has eigenvalues $n_\sigma(\mathbf{k}), n = 0, 1, 2, ...$ and eigenvectors defined as

$$a_\sigma(\mathbf{k})|n_\sigma(\mathbf{k})\rangle = \sqrt{n_\sigma(\mathbf{k})}|n_\sigma(\mathbf{k}) - 1\rangle, \quad a^*_\sigma(\mathbf{k})|n_\sigma(\mathbf{k})\rangle = \sqrt{n_\sigma(\mathbf{k})}|n_\sigma(\mathbf{k}) + 1\rangle.$$

Indeed,

$$|n_\sigma(\mathbf{k})\rangle = [[a^*_\sigma(\mathbf{k})]^{n_\sigma(\mathbf{k})}/\sqrt{n_\sigma(\mathbf{k})!}]|0\rangle.$$

The eigenvector of the radiation Hamiltonian given as equation (III) is a tensor product of such states, i.e.,

$$|\cdots n_\sigma(\mathbf{k})\cdots\rangle = \prod_{\mathbf{k},\sigma} |n_\sigma(\mathbf{k})\rangle \qquad (IV)$$

with the energy eigenvalues

$$E = \sum_{\mathbf{k},\sigma} \hbar\omega_k(n_\sigma(\mathbf{k}) + 1/2). \qquad (V)$$

The interpretation of these equations is a straight forward generalization from one harmonic oscillator to a superposition of independent oscillators, one for each radiation mode $(\mathbf{k}, \sigma)$. $a_\sigma(\mathbf{k})$ operating on the state (IV) will render occupational numbers unchanged. Indeed, we have

$$|a_\sigma(\mathbf{k})|\cdots n_\sigma(\mathbf{k})\cdots\rangle = \sqrt{n_\sigma(\mathbf{k})}|\cdots n_\sigma(\mathbf{k}) - 1\cdots\rangle \qquad (VI).$$

Correspondingly, the energy (V) is reduced by $\hbar\omega_k = hc|\mathbf{k}|$.

We interpret $a_\sigma(\mathbf{k})$ an "annihilation operator" which annihilates one photon in the model $(\mathbf{k}, \sigma)$, i.e. with momentum $\hbar\mathbf{k}$, energy $\hbar\omega_k$ and linear polarization vector $\mathbf{u}_{\mathbf{k}\sigma}$. Similarly, $a^*_\sigma(\mathbf{k})$ is interpreted as a "creation operator" of such a photon. We have

$$|a_\sigma(\mathbf{k})|\cdots n_\sigma(\mathbf{k})\cdots\rangle = \sqrt{n_\sigma(\mathbf{k})}|\cdots n_\sigma(\mathbf{k}) + 1\cdots\rangle \qquad (VII).$$

The state of the lowest energy of the radiation field is the "vacuum state" $|0\rangle$ in which all occupational numbers $n_\sigma(\mathbf{k})$ are zero. In lieu of (V), this state has energy $\frac{1}{2}\sum_{\mathbf{k},\sigma} \hbar\omega_k$.

Quantum field theory works only for the systems for which the zero-point energy of the radiative field cancels. According to Dirac, *for "many cases", this infinite energy of vacuum cancels out when physically meaningful quantities are calculated. So, we "assume" $H_{rad} = \sum_{\mathbf{k},\sigma} \hbar\omega_k a^*_{\mathbf{k}\sigma}a_{\mathbf{k}\sigma}$. A big question here is what are "physically meaningful quantities". A serious most question is what about energy? As we discussed in Section 1, the concept of energy is invalid.*

Remark 21 *Later, quantum field physicists took this problem seriously and Tomonaga-Schwinger presented a more general less ad hoc answer to this divergence problem which they called "renormalisation problem". Still this is far from a complete solution and all they did was to replace "many cases" by "all known cases". Mathematical point of view suggests that all of this implies that the fundamental idea of quantization a la Dirac is problematic and unattainable.*

The eigenvalues of this operator H_{rad} are $E = \sum_{\mathbf{k},\sigma} \hbar\omega_k n_\sigma(\mathbf{k})$. The momentum operator is $\mathbf{P} = \sum_{\mathbf{k},\sigma} \hbar\mathbf{k}(a^*_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma}) = \sum_{\mathbf{k},\sigma} \hbar\mathbf{k}(N_\sigma(\mathbf{k}))$ and its eigenvalues are $\sum_{\mathbf{k},\sigma} \hbar\mathbf{k}(n_\sigma(\mathbf{k}))$.

In conclusion, the following picture of the electromagnetic field emerges: It consists of photons each of which has energy $\hbar\omega_k$ and momentum $\hbar k$: $n_{\mathbf{k}\sigma}$ is the number of photons with momentum $\hbar\mathbf{k}$.The polarization is given by the vector $\mathbf{u}_{\mathbf{k}\sigma}$. Annihilation operator $a_{\mathbf{k}\sigma}$ decreases the number of photons with the momentum $\hbar\mathbf{k}$ by one and the creation operator $a^*_{\mathbf{k}\sigma}$ increases the number of photon with the momentum $\hbar\mathbf{k}$ by one.

7.2.4 The speed of Dirac's photons?

After all of this impressive applied mathematics, a simple common sense question remains to be answered. *Why Dirac's photons have anything to do with Planck-Einstein's photons? Why Dirac's photons move with speed c? Why the rest mass of Dirac's photon is zero. To be more forceful, we wonder what about the inconsistency of the Planck-Einstein's photon theory.* Does this not imply that this "impressive" work of Dirac is just a game in applied mathematics?

Going back to the question of the speed of Dirac's photons, this question arises from the lack of connections between Planck-Einstein's photons and Dirac's photons. Planck-Einstein's photons did not come from Fourier analysis at all while Dirac's photons are given birth through Fourier analysis of harmonic oscillators. Putting aside this intrinsic problem, we still have a serious mathematical problem. Fourier analysis was not pure mathematics but engineering mathematics and still pure mathematicians have some serious doubt about this expansion. The convergence issue of this expansion is still substandard compared with other series expansions of mathematical analysis. Moreover, the uniqueness of the expansion for the converging cases is negative. To find out this sad conclusion, George Cantor developed set theory which turned out to be inconsistent as he himself discovered. This disappointing conclusion makes the application of this expansion impossible for any serious applications. In the obscurity of applied mathematics, this transformation has some kind of "role". But it also is a fact that for many engineering applications, this expansion tends to give the approximation they need for practical purposes. But if this expansion is acceptable in theoretical physics is a different story.

7.3 Dirac's "quantization" of Schrödinger's wave equation (the second quantization)

Without knowing any of these issues with his quantization of electromagnetic fields, Dirac went on to apply the idea of quantizing Maxwell's em wave equations to the Schrödinger's wave equations. If this was Dirac's final answer to the frustrating problem of the failure to make Schrödinger's wave equation relativistic is not quite clear.

It is quite clear that such project requires conceptual understanding which is a little deeper than Dirac expected. Maxwell's wave equations and Maxwell's em field equations are different things. The former is a consequence of the latter which are modality; therefore em waves are not reality but modality.

Dirac quantized L-bounded em field and from this he came up with the quantization of em waves as a system of harmonic oscillators. Unfortunately we can not platy the same game here for Schrödinger's wave equations because there is no such thing as the field theory which supports all Schrödinger's waves. So, he ended up with quantizing Schrödinger's wave equation directly.

Moreover, Schrödinger's wave equation came from the special theory of relativity of Einstein through the highly questionable de Broglie relation applied to the classical Hamilton's equation of motion. As we know, Schrödinger's wave equation failed to be relativistic. So, the validity of the quantization of such highly controversial equation is highly questionable and this makes the relevance of the quanta of Dirac for Schrödinger's wave equations highly questionable at best.

Nevertheless, in what follows, we will discuss how Dirac's second quantization went.

Consider Schrödinger's equation

$$-(\hbar/i)(\partial\Psi/\partial t) = -\hbar^2/2m\nabla^2\Psi + V(\mathbf{x})\Psi$$

for a particle in a potential $V(\mathbf{x})$. Let Ψ_n and E_n be the eigenvectors and eigenvalues of the operator $(-\hbar^2/2m)\nabla^2 + V(\mathbf{x})$. This is to say

$$\left(-\hbar^2/2m)\nabla^2 + V(\mathbf{x})\right)\Psi_n = E_n\Psi_n.$$

The "Fourier expansion" of the wave function is,

$$\Psi(\mathbf{x}, t) = \sum_n b_n(t)\Psi_n(\mathbf{x}).$$

Substituting this to the Schrödinger's equation yields

$$(db_n)/dt = (-1/h)E_n b_n.$$

The expected value of energy is

$$H = \int d^3x\Psi^*(\mathbf{x}, t)\left[(-\hbar^2/2m)\nabla^2 + V(\mathbf{x})\right]\Psi(\mathbf{x}, t).$$

Putting all of these equations together and the orthogonality of Ψ_n we have

$$H = \sum_n E_n b_n^* b_n.$$

This is the Hamiltonian for a collection of harmonic oscillators with frequencies $E_n/\hbar$. If we consider b_n as an operator then b_n^* can be considered as the adjoint of b_n, in, symbols b_n^+ Under the commuting relations

$$[b_n, b_{n'}] = [b_n^+, b_{n'}^+] = 0, \qquad [b_n, b_n^+] = 0$$

from Heisenberg's equation

$$-\hbar(db_n/dt)/i = [b_n, H]$$

we can derive Fourier version of the Schrödinger equation as planned.

In this way, a Schrödinger's wave equation which is obtained from Hamiltonian is represented by an infinite system of "oscillating particles", as Dirac planned.

Remark 22 *Clearly, Dirac is using the claimed "empirical equivalence" between Heisenberg-Jordan's quantum mechanics and Schrödinger's quantum mechanics here. Putting aside the validity of this claimed equivalence, it is a highly irregular mathematical and logical exercise to mix up two different theories under empirical equivalence in general. This concern is material especially because the empirical equivalence here is of probabilistic nature.*

Remark 23 *The real problem here is that as Kuhn pointed out, the relative frequency converges only at limit and so probabilistic prediction is not empirical at all as we can not repeat the same experiment infinitely many times. The lack of disciplined thinking is sticking its ugly head again here.*

The operators $b_n^+ b_n$ have the eigenvalues $N_n = 0, 1, 2, ...$, indicating that any natural number of particles may occupy the eigenstate Ψ_n. The eigenvalue of H then is $E = \sum_n E_n N_n$.This theory obeys Bose-Einstein statistics and these particles are called *Bosons*.

This theory excludes particles which obey Fermi-Dirac statistics. These particles are called *Fermions*. A minor change of the theory above will derive a theory of Fermions, we keep Hamiltonians as $H = \sum_n E_n b_n^* b_n$. We expect the Heisenberg equation of motion to yield $db_n/dt = -E_n b_n/\hbar$. The only change involved is the commuting relations

$$[b_n, b_{n'}] = [b_n^+, b_{n'}^+] = 0, \qquad [b_n, b_n^+] = 0$$

to the commuting relations

$$[b_n, b_{n'}]_+ = [b_n^+, b_{n'}^+]_+ = 0, \qquad [b_n, b_n^+]_+ = \delta_{n,n'}$$

where $[A,B]_+ = AB + BA$. Now we have

$$(-\hbar/i)(db_n)/dt = [b_n, H] = \sum_m E_m\{b_n b_m^+ b_m - b_m^+ b_m b_n\} = \sum_m E_m \delta_{nm} b_m = E_n b_n.$$

So, we have obtained Heisenberg equation of motion. Note that

$$(b_n^+ b_n) b_n^+ b_n = b_n^+(1 - b_n^+ b_n) b_n = b_n^+ b_n - b_n^+ b_n b_n^+ b_n = b_n^+ b_n.$$

If λ is an eigenvalue of $b_n^+ b_n$ then

$$b_n^+ b_n |\lambda\rangle = \lambda|\lambda\rangle \qquad b_n^+ b_n b_n^+ |\lambda\rangle = \lambda^2|\lambda\rangle = \lambda|\lambda\rangle.$$

Thus $\lambda^2 = \lambda$. This is to say $\lambda = 1$ or $\lambda = 0$. This means that *at most one particle can occupy the eigenstate* Ψ_n. We may write $|n\rangle$ to denote this eigenstate. This theory obeys Fermi-Dirac statistics.

To express all of this on λ, we may write $b_n^+ b_n |N_n\rangle = N_n |N_n\rangle$ where $N_n = 0, 1$. Now we have

$$b_n^+ b_n b_n^+ |N_n\rangle = b_n^+(1 - b_n b_n^+)|N_n\rangle = (1 - N_n) b_n^+ |N_n\rangle.$$

This implies that $b_n^+|N_n\rangle$ is an eigenvector of $b_n^+ b_n$ with the eigenvalue $1-N_n$. It can only differ from $|1-N_n\rangle$ only by a constant. We write $b_n^+|N_n\rangle = C_n|1-N_n\rangle$. The constant C_n can be evaluated by taking the inner product of $b_n^+|N_n\rangle$ with itself.

$$\langle b_n^+|N_n\rangle, b_n^+|N_n\rangle\rangle = (1 - N_n) = C_n^* C_n.$$

Thus we have $C_n = \theta_n\sqrt{1 - N_n}$ where θ_n is a phase factor of modulus unity. This leads to

$$b_n^+|N_n\rangle = \theta_n\sqrt{1 - N_n}|1 - N_n\rangle \qquad b_n|N_n\rangle = \theta_n\sqrt{N_n}|1 - N_n\rangle.$$

In summary we have

1. For bosons:

$$b_n|\cdots, N_n, \cdots\rangle = \sqrt{N_n}|\cdots, N_n{-}1, \cdots\rangle b_n^+|\cdots, N_n, \cdots\rangle = \sqrt{N_n}|\cdots, N_n{+}1, \cdots\rangle$$

2. For Fermions:

$$b_n|\cdots, N_n, \cdots\rangle = \theta_n\sqrt{N_n}|\cdots, 1{-}N_n, \cdots\rangle \qquad b_n^+|\cdots, N_n, \cdots\rangle = \theta_n\sqrt{1 - N_n}|\cdots, 1{-}N_n, \cdots\rangle$$

where $N_n = 0, 1$. In both cases, b_n is annihilation operator and b_n^+ is a creation operator.

One more question remains to be answered. Why Dirac started with second quantizing Schrödinger's wave equations? Why he did not start directly with Hamiltonian equations? Was it because Hamiltonian equations are just classical equations of energies while Schrödinger's wave equations are "relativistic"? If so, it is odd because Schrödinger was panicking because he could not make his wave equations fully relativistic despite that he used STR (in the form of de Broglie relation) to convert Hamiltonian equations to his wave equations

Dirac wanted to quantize "energy fields in general" in a similar way to what he did to electromagnetic fields so that the same theoretical frame applies to energies in general expressed as Hamiltonians. In this project, *he treated waves as fields (mediums), as there is no general fields equations to general energy. This is a major difference between quantization of em energy and Hamilton energy.*

After all, as we have been discussing, the concept of energy is a modal concept and it is not defined properly. We, in fact know, very little about what energy is.

7.4 Interactions of quantum particles

We can add the Hamiltonians for several free particle fields and introduce appropriate interaction terms to study *interacting particle fields*. The most common such interaction is that of photons with charged particles. *We use the theory of second quantization to represent a charged particle field by the following Hamiltonian:*

$$\int d^3x\Psi^+(\mathbf{x},t)\left[(-\hbar^2/2m)\nabla^2+V\right]\Psi(\mathbf{x},t).$$

The quantized electromagnetic field is represented by the following radiation (photon) Hamiltonian:

$$\int d^3x\frac{1}{8\pi}(E^2+B^2).$$

The interaction of these two fields will be obtained by adding these two Hamiltonians and prescribing the following replacement:

$$\frac{\hbar}{i}\nabla\Longrightarrow\frac{\hbar}{i}\nabla-\frac{e}{c}\mathbf{A}(\mathbf{x}).$$

This leads to

$$H=\int d^3\mathbf{x}\Psi^+(\mathbf{x},t)\left[(-\hbar^2/2m)\left|(\hbar/i)\nabla-(e/c)\mathbf{A}(\mathbf{x})\right|^2+V\right]\Psi(\mathbf{x},t)+\int d^3x(E^2+B^2)/8\pi=H_P+H_{rad}+H_I$$

where

$$\int d^3x\Psi^+(\mathbf{x},t)\left[-(-\hbar^2/2m)\nabla^2+V\right]\Psi(\mathbf{x},t)=\sum_n E_nb_n^+b_n$$

which is the *particle Hamiltonian*,

$$H_{rad}=(1/8\pi)\int d^3\mathbf{x}(E^2+B^2)=\sum_{k,\sigma}\hbar\omega_ka^+_{\mathbf{k},\sigma}a_{\mathbf{k},\sigma}$$

is the *radiation field Hamiltonian*, and

$$H_I=\int d^3x\Psi^+(\mathbf{x},t)\left[-(\hbar^2/imc)\mathbf{A}\cdot\nabla^2+(e^2/2mc^2)A\right]\Psi(\mathbf{x},t)$$

is the *interaction Hamiltonian*. We can divide H_I into a part H' proportional to A and a part A' proportional to A^2 such that $H_I = H' + H$". Expanding A and Ψ in terms of $a_{\mathbf{k},\sigma}$ and b_n gives

$$H' = \sum_{\mathbf{k},\sigma}\sum_n\sum_{n'}\left[M(\mathbf{k},\sigma,n,n')b_n^+b_{n'}a_{\mathbf{k},\sigma} + M(-\mathbf{k},\sigma,n,n')b_n^+b_{n'}a^+_{\mathbf{k},\sigma}\right]$$

and

$$\begin{aligned} H" \quad = \quad & \sum_{\mathbf{k}_1,\sigma_1}\sum_{\mathbf{k}_2,\sigma_2}\sum_n\sum_{n'} M(\mathbf{k}_1,\sigma,\mathbf{k}_2,\sigma_2,n,n')a_{\mathbf{k}_1,\sigma_1}a_{\mathbf{k}_2,\sigma_2} + M(\mathbf{k}_1,\sigma_1,-\mathbf{k}_2,\sigma_2,n,n')a_{\mathbf{k}_1,\sigma_1}a^+_{\mathbf{k}_2,\sigma_2} \\ & +M(-\mathbf{k}_1,\sigma_1,\mathbf{k}_2,\sigma_2,n,n')a^+_{\mathbf{k}_1,\sigma_1}a_{\mathbf{k}_2,\sigma_2} + M(-\mathbf{k}_1,\sigma_1,-\mathbf{k}_2,\sigma_2,n,n')a^+_{\mathbf{k}_1,\sigma_1}a^+_{\mathbf{k}_2,\sigma_2} \end{aligned}$$

where

$$M(\mathbf{k},\sigma,n,n') = \sqrt{2\pi\hbar c^2/\Omega\omega_k}\int d^3\mathbf{x}\Psi_n^*\left[-e\hbar e^{i\mathbf{k}\cdot\mathbf{x}}/imc\mathbf{u}_{k,\sigma}\cdot\nabla\right]\Psi_{n'}$$

and

$$\begin{aligned} & M(\mathbf{k}_1,\sigma,\mathbf{k}_2,\sigma_2,n,n') \\ = \quad & \sqrt{2\pi\hbar c^2/\Omega\omega_k}\sqrt{1/\omega_{\mathbf{k}_1}\omega_{\mathbf{k}_2}}\int d^3\mathbf{x}\Psi_n^*\left[(-e\hbar/2mc^2)e^{i\mathbf{k}\cdot\mathbf{x}}\mathbf{u}_{\mathbf{k}_1,\sigma_1}\cdot\mathbf{u}_{\mathbf{k}_2,\sigma_2}e^{i(\mathbf{k}_1+\mathbf{k}_2)\cdot\mathbf{x}}\right]\Psi_n. \end{aligned}$$

The part of the Hamiltonian $H_p + H_{rad}$ can be considered the unperturbed part with eigenvectors and eigenvalues .

$$|\cdots N_n\cdots\rangle_p\,|\cdots n_{\mathbf{k},\sigma}\cdots\rangle_{rad}\,,\ \sum_n E_nN_n + \sum_{\mathbf{k},\sigma}\hbar\omega_{\mathbf{k}}n_{\mathbf{k}\sigma}$$

respectively.

The interaction Hamiltonian H_I induces transitions between these states as follows:

1. the term $b_n^+b_{n'}a_{\mathbf{k},\sigma}$ in $H^{'}$: (1) annihilates a photon of momentum $\hbar\mathbf{k}$ and polarization σ, (2) annihilates a particle in state $|n'\rangle$, (3) creates a particle in state $|n\rangle$.

2. the term $b_n^+b_{n'}a_{\mathbf{k},\sigma}$ in $H^{'}$: (1) creates a particle in state $|n\rangle$, (2) annihilates a particle in state $|n'\rangle$, (3) annihilates a photon of momentum $\hbar\mathbf{k}$ and polarization σ.

3. the term $b_n^+b_{n'}a_{\mathbf{k},\sigma}$ in $H^{''}$: (1) creates a particle in state $|n'\rangle$, (2) annihilates a particle in state $|n\rangle$, (3) annihilates a photon of momentum $\hbar\mathbf{k}_1$ and polarization σ_1, (4) annihilates a photon of momentum $\hbar\mathbf{k}_2$ and polarization σ_2.

4. the term $b_n^+b_{n'}Ma_{\mathbf{k}_1,\sigma_1}a^+_{\mathbf{k}_2,\sigma_2}$ in $H^{''}$: (1) creates a particle in state $|n\rangle$, (2) annihilates a particle in state $|n'\rangle$, (3) annihilates a photon of momentum $\hbar\mathbf{k}_1$ and polarization σ_1, (4) create a photon of momentum $\hbar\mathbf{k}_2$ and polarization σ_2.

5. the term $b_n^+ b_{n'} M a^+_{\mathbf{k}_1,\sigma_1} a_{\mathbf{k}_2,\sigma_2}$ in $H^{''}$: (1) creates a particle in state $|n\rangle$, (2) annihilates a particle in state $|n'\rangle$, (3) create a photon of momentum $\hbar\mathbf{k}_1$ and polarization σ_1, (4) annihilates a photon of momentum $\hbar\mathbf{k}_2$ and polarization σ_2.

6. the term $b_n^+ b_{n'} M a^+_{\mathbf{k}_1,\sigma_1} a_{\mathbf{k}_2,\sigma_2}$ in $H^{''}$: (1) creates a particle in state $|n\rangle$,(2) annihilates a particle in state $|n'\rangle$, (3) create a photon of momentum $\hbar\mathbf{k}_1$ and polarization σ_1, (4) create a photon of momentum $\hbar\mathbf{k}_2$ and polarization σ_2.

7.5 Renormalization

7.5.1 Renormalization in fluid dynamics

Stokes, the 19th century fluid mechanist introduced the *renormalized mass* m for a mass m_0 moving in a fluid with speed v as $m = m' + m_0$. This m' is a constant determined by the geometry of the system expressed by the boundary condition of the system and the density of the fluid. With this he decided that the kinetic energy of this body is $K = mv^2/2$. The extra mass m' is nothing but the manifestation of kinetic energy of the fluid pushed by the moving body. This development is an interesting violation of the principles behind particle dynamics and fluid dynamics which are completely different theories, which conceptually contradict each other. In particle dynamics, as Newton set it, *there is no geometric dimension to particles*. They are all *point masses*. If so, there must be no resistance from the fluid when such point mass moves and the entire concept of reorganized mass is meaningless. Anyhow, fluid dynamics is not physics. It is utilitarian engineering which appeared as a branch of engineering. So, the concept of renormalized mass makes no theoretical sense. But this is how the world of applied science is and it is the duty of pure theoreticians to pay due attention to this kind of intrinsic change of paradigm.

7.5.2 Renormalization in classical electrodynamics

Thomson observed an analogy between the motion of a solid body through an incompressible fluid and motion of a body through its own electromagnetic field (or through *aether*). Through this analogy, he introduced the notion of *electromagnetic mass* of a charge. There were two reasons behind this concept.

1. If a charge which also is a mass moved through the aether which is a "fluid", according to the fluid dynamics, it should acquire an additional mass as per Thomson.

2. According to the Maxwell's theory, a moving charge creates a magnetic field which possesses energy coming from the force which accelerates the charge from speed 0 to v. This extra energy manifests as an additional inertia possessed by the charge, which we observe as the additional mass, which is called *electromagnetic mass* denoted by m'.

For a spherical charge of radius r and charge e, Thomson calculated the kinetic energy as

$$K = \left(m + 4\mu e^2/15r\right) v^2/2$$

where μ is the electric permittivity of the medium and v is the speed of the body. The electromagnetic mass is $4\mu e^2/15r$. In vacuum where $\mu = \mu_0 = 1/\epsilon_0 c^2$ where ϵ_0 is the electric permittivity, for a uniformly charged sphere of radius r, the electromagnetic mass is

$$m' = 2e^2/3rc^2.$$

Clearly all of this is based upon the assumption of aether which is a material manifestation of electromagnetic field which we now know is *counter factual modality* which Thomson did not understand.

This idea of Thomson was further developed by his successor Lorentz who over wrote Thomson on the most fundamental understanding of aether. Thomson believed the Newtonian "materiality" of aether but for Lorentz despite that aether exerts force upon charges, the charges do not exerts force upon the aether. In this way, Lorentz's aether violates the third law. Despite his rejection of Einstein's anti-Newtonian physics, namely relativity theory, Lorentz also rebelled against Newton's classical view. His aether was anti-Newtonian aether.

Remark 24 *We pointed out a closely related issue in many places that the concept of electromagnetic field (or any force field more generally) violates the third law. Force field exerts force upon physical body but not the other way around. Indeed this is why Newton rejected the concept of any force field in the end.*

To Lorentz, an "electron" is a finite distribution of charge with finite boundaries. So, all-pervasive aether and electromagnetic field must be inside charges. This view leads to an outstanding distinction between external fields and internal fields. The external field is created by the external sources and the self field is created by the interaction between the internal charge distribution and the aether. The way how this interaction manifests depends upon the motion of electrons through the stationary aether. If there is no outside charge, the electron must move exactly like a point charge with a constant speed. This excludes spin of such electrons.

Upon this development and the conservation law, Lorentz obtained the force an electron experiences when moving through the aether (or field) as

$$F = (2e^2/3rc^2)(d^3x/dt^3)$$

under the assumption that the distribution of the charge is spherically symmetric. Interestingly Thomson's theory and Loorentz's theory agree on the electromagnetic mass of a spherical charge which is

$$m' = (2e^2/3rc^2).$$

However we must admit that this argument of Lorentz is invalid as it ignores the necessity to reduce a charge to a point charge to consider the electromagnetic force an electron experiences in the aether. It appears to be a difficult task to put one feet on pragmatism and the other on idealism.

7.5.3 Kramer's renormalization

Kramer's criticism on the standard quantum electrodynamics was that they start with the un-renormalized Hamiltonians and try to remove the problem of divergent terms by absorbing them into later introduced renormalized mass or renormalized charge. He criticized this practice as a *"band aid solution"*. He stressed that we must quantize the correct classical picture. He pointed out that as Thomson and Lorentz already dealt with the renormalization of mass, we should quantize their theory rather. In this sense he strongly criticized Dirac's quantization of un-renormalized (not having electromagnetic mass) Maxwell's electrodynamics.

According to Kramer, the basic equation in the spirit of Lorentz is

$$m_0(d^2x/dt^2) = \mathbf{K} + \mathbf{F}_{ext} + \mathbf{F}_{self}$$

where m_0 is the material (mechanical) mass, F_{ext} is the external force, F_{self} is the self-force and K is the non-electromagnetic force. According to the Lorentz theory,

$$\mathbf{F}_{self} = -\frac{2}{3c^2}\frac{d^2x}{dt^2}\iint d^3x d^3x' \frac{\rho(x)\rho(x')}{|x-x'|} + \frac{2e^2}{3rc^2}\frac{d^3x}{dt^3} + g\frac{re^2}{c^4}\frac{d^4x}{dt^4} + \cdots$$

where r is the radius of the charge distribution $\rho(x)$. The term

$$-\frac{2}{3c^2}\iint d^3x d^3x' \frac{\rho(x)\rho(x')}{|x-x'|}$$

is the electrostatic self-energy of the charge distribution $\rho(x)$ and it is the electromagnetic mass m'. This term diverges as $r \to 0$. Now we have the experimental mass $m_{\text{exp}} = m_0 + m'$,and we have

$$(m_{\text{exp}} - m')\frac{d^2x}{dt^2} = \mathbf{K} + \mathbf{F}_{ext} - m'\frac{d^2x}{dt^2} + \frac{2}{3c^2}\frac{d^3x}{dt^3} + g\frac{re^2}{c^4}\frac{d^4x}{dt^4} + \cdots$$

Therefore,

$$m_{\text{exp}}\frac{d^2x}{dt^2} = \mathbf{K}+\mathbf{F}_{ext}+\frac{2}{3c^2}\frac{d^3x}{dt^3}+g\frac{re^2}{c^4}\frac{d^4x}{dt^4}+\cdots = \mathbf{K}+e\left(\mathbf{E}_{ext} + \frac{[\mathbf{v}\times\mathbf{B}_{ext}]}{c}\right)+\frac{2e^2}{3c^2}\frac{d^2x}{dt^2}.$$

For Kramer, this was a perfect solution as it contains directly observable experimental mass and observable external field.

His plan was to quantize this equation to construct a satisfactory quantum electrodynamics. Of course in his equation above, m_{exp} contains m', the model dependent divergent electromagnetic mass.

However he observed that this problematic divergence is somehow contained. It indeed is hidden in the experimental mass "which we always observe finite". Unfortunately this is yet another example of highly debatable issue of covering up the deficiency of theory using empiricism. We discussed this issue in subsection *5.4 Empirically most verified theory?* and elsewhere.

After all, Kramer's program included the following elements:

(1) Construction of a Hamiltonian for classical spherical charge of radius r interacting with a given external electromagnetic field.

(2) Separation of electromagnetic field into a self-field and an external field.

(3) Separation of electromagnetic mass and mechanical mass.

(4) The elimination of the structure-dependent terms through a series of canonical transformations as absorption into empirical parameters without such terms appearing as dynamic variables.

(5) On the structure-independent formulation, in principle, limit $r \to 0$ should converge.

(6) The second quantization of a structure-independent formulation should gives us a satisfactory quantum theory of radiation.

Kramer's program was never completed. However, one of the most important and interesting contribution of Kramer was the concept of *cut off.* In the Fourier decomposition of the electromagnetic field, only Fourier components which have the wave length $\lambda > r$ were considered. This technically offers a dramatically simplified *"mathematical treatment"* of diverging terms. However it is at the cost the point charge assumption which is essential for all dynamics. *This can be considered as a pragmatic convention which violates both theoretical physics and mathematics. Theoretical physics is not engineering.* It is interesting that theoretical physics diverged from pure mathematical science in parallel to the development of em theory. This theory thrived because of its direct connection to electrical engineering.

Electrodynamics is the first unacceptable combining of particle physics and continuum physics. It is notable that fluid dynamics steps out of the boundary of what we call "theoretical physics" started by Newton who for totally legitimate reason rejected the concept of continuum force field. *This discipline considered continuum matter, called fluid called "aether", and tried to apply the concept of force and motion upon it (the continuum matter).* The end product is logically and conceptually incoherent "engineering science" which *"works" in engineering only. However, at least fluid dynamic is conceptually coherent in its avoiding of particles.*

The above mentioned peculiarity of classical electrodynamics leads us to understand why electrodynamics, either classical or quantum produced so much confusion. The historic fact that Thomson adopted his renormalization problem for classical electrodynamics from Stokes's original renormalization problem in classical fluid dynamics clearly supports this view. It now is clear the reason why electrodynamics classical or quantum had hard time is due to this mixing up of continuum and particles. Particle dynamics and fluid dynamics do not work together well as pure mathematical science due to the disjointness of "continuum fluid" and "discrete particles". Certainly there is no point matter in fluid dynamics. Stokes's renormalized mass appears not inside a point mass but inside a mass which has geometric continuum shape and geometric dimension. It is the summation of all effects the massive body gets from the fluid and it appears inside the massive body. Original Coulomb electromagnetism theory, following the philosophy of Newton, considered only point charges. *In short, particle physics and continuum physics are entirely different categories as Newton and Coulomb rightly warned.*

Remark 25 *There is one more important factor to be discussed regarding the peculiarity of electrodynamics. Despite that this theory started with almost identical beginning of Newtonian dynamics, it under the strong influence of Heaviside and Hertz upon Maxwell was twisted into the theory of force field, namely electromagnetic field, which is a serious category error, as force field is not a physical reality but it is a counter-factual modality. The spacial distribution of force per a unit charge is not a physical reality. It is impossible physically to place unit charge everywhere in the space.*

Moreover, if we want to bring in continuum physical bodies, we must mathematically reconcile such constructs and particles. This most important issue of physics was totally ignored as it was too much of mathematical work for "theoretical physicists". Particle systems, no matter how much packed, are not even dense. So they are not continuum as we can always find a gap in between two particles in physical reality. Real continuum is dense which means that in between two particles, we can always find another particle. The continuum of original "continuum physics", however, is real continuum. This is why we can not integrate "particle physics" and "continuum physics". The other way of saying is simply that *geometric points in 3D space and physical point particles are "not the same thing" at all.* It is unfortunate that no theoretical physicists seem to understand this and no mathematicians bother to discuss the issue.

Remark 26 *On mathematicians side, this is a challenging problem as it shows that the mathematics of continuum as we have now is not usable for theoretical physics. Mathematicians do not see it in this way as it also means that mathematics has to change to work with physics. Application of mathematics is now reduced to something like information science and probability theory. The most vital part of mathematics disappeared from the research front. The four colour conjecture and the Fermat last theorem became the biggest achievement simply means mathematics has degenerated to popular science. Nobody in mathematics seem to have courage to stand up and say enough is enough. In physics, movement for change to better has been going on and now gaining strong momentum. There is no more forests left in mathematics We are just looking at trees. We are just looking for already fallen autumn tree leaves. Is there any mathematics which can unite the particle physics and continuum physics. Theoretical physicists have challenged and get stuck. No mathematician pays attention to this important problem. The goal of mathematics is not just solving famous open problems as Perelman said.*

7.5.4 Tomonaga-Schwinger renormalization

In relativistic theory which starts with relativistic wave equations, the negative energy states are not available for constructing a wave packet to represent a positive energy electrons as they are full. It is as if the electron has a finite size approximately equal to its Compton length. *This limit on the size of the electron wave packet makes the divergence of self-energy logarithmic instead of linear. For logarithmically divergent integrals, we can separate diverging components from meaningful converging components.* Tomonaga used this fact to resolve the issue of diverging terms in QED. All of this indicates that Dirac's formalism misses something of "physical importance". Beyond this apparent success of Tomonaga-Schwinger, it might be more desirable to build a formalism which looks after this important issue from the start. Band aid solution does not lead very far.

Consider $f(x) = \int_1^\infty dy/(x+y)$. As y gets large, the integral becomes like $1/y$ and approaches to zero. However, not fast enough to keep the integral from diverging logarithmically. So, $f(x)$ is divergent at all x. In QED, the variables x and y represent energy variables the integral represents one of the sums over virtual states to be considered. Now

$$\bar{f}(x) = f(x)-f(0) = \int dy[1/(x+y)-1/y] = \int dy[(y-x-y)/y(x+y)] = -x\int_1^\infty dy/y(x+y).$$

Despite the divergence of $f(x)$ and $f(0)$, the difference $\bar{f}(x)$ converges. This is to say that the diverging integral $f(x)$ can be separated into two parts $f(x) = A + \bar{f}(x)$ where A is the infinite constant $f(0)$. It is the converging part $\bar{f}(x)$ which carries physical meaning.

In QED, we also have linearly diverging integrals. In what follows, we will show that such divergence can also be separated into diverging parts and converging parts. Consider $g(x) = \int ydy/(x+y)$.The integrand is a constant as $y \to \infty$. Thus the integral is linearly divergent. Now

$$g(x)-g(0) = \int ydy[1/(x+y)-1/y] = -x\int ydy/(x+y) = -x\int dy/(x+y) = -xf(x) = -x[A+\bar{f}(x)].$$

Though Ax diverges, $x\bar{f}(x)$ is converges. Therefore

$$g(x) = g(0) - Ax - x\bar{f}(x) = g(0) - A\bar{f}x + \bar{g}(x) = B - Ax + \bar{g}(x)$$

where B is divergent and integrals $g(0)$ and $\bar{g}(x) = -x\bar{f}(x)$ is convergent. We have separated the physically meaningful convergent term $\bar{g}(x)$ from divergent integral $g(x)$.

As the only divergence "we meet" in QED are either logarithmic or linear, these prototypes we presented above serves as a general scheme to isolate the divergent components to extract meaningful information from the theory.

In what follows we present an interesting and "moving" history of active and positive interaction between different approaches which is unique in the history of theoretical physics.

(1) Under the influence of Heisenberg, Tomonaga studied the effect of a field upon its own source which is the first explicit attempt to address the issue of the violation of the third law by the introduction of force field. Despite its positive side, this created a vicious circle which was inevitable. To be precise, we still have no clear answer to this dilemma. This is a deep philosophical issue which can not be resolved by equational calculus of QED. Unfortunately for QED experts the issue is far too philosophical and they pay no attention at all, which is one of the major cause of the crisis physics is in. This issue was revisited later by Feynman who discussed a similar concern with Wheeler.

(2) Anyhow in this context, Tomonaga adopted Sakata's proposal that a conjectured field, namely "C-meson field", might interact with the electron which created the quantum field and produce a negative infinite electron mass to cancel the positive infinite mass produced by the electromagnetic field. To push this idea, Tomonaga developed a theory for a completely covariant QED based upon "super-many-time theory" and turned to Dancoff's theory of the elastic scattering which indicated the logarithmic divergence of energy in Dirac's relativistic theory with spinners. This theory of Tomonaga was a generalization of Dirac-Fock-Podolowsky's "many time theory" which treated space time theory of each particle in an interacting system separately, contrary to the formalism based upon Hamiltonian theory.

1. Tomonaga described electrons and photons as quantum fields with infinite number of degrees of freedom, he had to assume a time for each space point.

2. Tomonaga then found an error in Dancoff's calculation on Dirac's electron. The then showed that the result was not infinite, it is finite. Thus there is no need for C-meson field. Which was a good relief for physics indeed.

3. Further study processes in QED such as elastic scattering lead Tomonaga to the conclusion that all diverging integrals in QED can be removed by covariant subtraction.

7.5.5 Feynmann's renormalization

The general result of Tomonaga did not deter Feynman from attempting a new and more ontologically desirable renormalization theory using Kramer's suggestions.

Feynman when he was a graduate student of Wheeler expressed his view that the cause of the infinite energy of electron could be removed simply by reformulating classical electrodynamics so that the field produced by the electron did not interact with the electron itself. *He was proposing to convert electrodynamics as a theory of delayed action at a distance, going back to the good old time of Gauss and Weber.*

However, he did not come to the realization that the concept of electromagnetic field is not a physical reality but a counter factual modality and this is the real cause of the confusion spread all over theoretical physics, Feynman rightly pointed finger at the role of force field which caused the divergence problem of the QED despite the short fall that the concept of force field is not in the do-main of physical reality. This view was already articulated by Newton 200 years before Feynman who rejected the usage of gravitational force field opposing the view of Leibniz.

It appears that the response from Wheeler to Feynman was a misleading one typical of contemporary theoretical physics of the time. First he pointed out that without self-interaction, there would be no radiative reaction so that a radiating charge could not loose energy or momentum. He also suggested that this could be resolved, at least in classical setting, by replacing the interaction which is normally delayed because of the finite speed of light, by an interaction which was half advanced and half retarded.

Clearly Wheeler did not know what he was talking about. If the interaction is delayed because of the finiteness of the speed of light, then how is it possible that a charge spread its electric effect all over the infinite space as the field theory tells us. It is this kind of total lack of understanding, compensated by the mindless symbolic calculation which destroyed theoretical physics.

Putting momentum aside, we have shown that the concept of energy is ill-defined. Moreover, it is a modality as it is defined as the "potential" to do work. When it comes to momentum, it is not clear how the absence of self- interaction will loose momentum. To begin with, classical mometum conservation law is assumed only for colliding masses with constant speed motion. It appears that the classical dynamics has been twisted into something else through a mountain of opportunistic conventions introduced to original theory which rejected them. We are saying that the real problem lies in the sad reality that the classical theories turned into logically inconsistent theories and the pinnacle of it was the electrodynamics. *It is a miracle if the quantization of such inconsistent theories can produce a desirable theory.*

In his attempt to create a quantum version of the above mentioned idea, Feynman realized that it was needed to keep track of particles having their own time variables. Therefore, he replaced the standard Hamiltonian method by the Lagrangian time-space integral. This developed into the path integral method. In classical theory, time and space are not physical entities. They are *metaphysical entities.* Physics is expressed upon these "untouchable" (by physics) metaphysical entities. Feynman certainly made the same categorical error as Einstein in violating this sacred status of the category of time. *As we know, Einstein made time altered by speed while speed is defined in terms of time.* It is a philosophical and logical trivia that once we violate this category hierarchy, we immediately encounter contradictions. Certainly, Feynman's theory ended up with the paradox of a matter moving backward in time "as its anti-matter".

What is more outstanding is that this "revolutionary result" was produced through the remarkably "revolutionary" process of *"quantization by replacing classical variables with quantum operators"* as Gordon-Klein started.

The next phase of Feynman was to go back to the old idea of delayed action at a distance as per Newton and Coulomb, with a "relativistic twist". He tried to redefine the classical electromagnetic interaction using the relativistic version of principle of the least action. This resulted in a relativistically invariant modification of electromagnetism theory which did not use the infinite self-mass.

For this project, he used a relativistic version of the cut off introduced by Kramer. As Feynman's theory is momentum based, he used "momentum cut off" instead. With the cut off set arbitrary large but finite number, dealing with infinity was excluded. For Feynman, cut off meant the separation of the lower momentum knowable world and the higher momentum unknowable world. For Kramer cut off meant the separation of the observable external world and unobservable internal world. This strange view of Kramer all came from the ill fated introduction of charges which are not point charges in violation of the most basic assumption of electrodynamics.

All of this nicety is qualified by the condition that the relativity theory is consistent. We have shown that this theory is hopelessly inconsistent destroying all modern theories which relied upon it. Certainly relativistic version of Feynman's quantum mechanics is all invalid.

Putting aside the validity issue, in the relativistic version of Feynman's theory, the "momentum based" qualification means nothing. It is because of the infamous momentum-energy equation of Einstein.

7.5.6 Dyson's unification

To sum up the whole development of the renoramlzation in QED, Dyson showed that Feynman's result is obtainable from Tomonaga-Schwinger formulation of QED. He proved that the renormalization of mass and charge will guarantee the finiteness QED corrections to any finite order of perturbation theory though it would not guarantee the convergence of the infinite series. His view was that renormalization is essentially associated with averaging out the large fluctuation of quantum fields which is equivalent to blurring exact point model of quantum field.

With this result of Dyson, the realistic aspects of Feynman's cutoff approach became obsolete. By taking the cut off to infinity, all high energy processes are taken cared of and all of their effects upon low energy oppresses are absorbed by redefining the parameters as Tomonaga did. We can no longer consider cutoff as the threshold energy at which the theory stops being valid and unknown new theories are required. *Taking the cutoff to the infinity implies that the theory is valid at any energy level. Ironically this implies that renormalization is not needed at all.*

After decades of neglect, the realistic aspect of cut off was attended again with the devastating conclusion that the taking cut off to infinity reduces the role of cut off to Tomonaga-Schwinger's formalistic method to deal with the infinity.

8 Quantum gravity theory: history

8.1 Classical field theory

Gravitational wave theory was started as an analogy to the em wave theory of Maxwell. The researchers in neither of the fields realize that a gravitational (or electric) force field which is a spacial distribution of gravitational force (or electric force) force per unit mass (or charge) is not a physical entity. It is a (counter factual) modality not a physical reality. They wrongly take it as physical entity like water for water waves, which is a physical reality. It is astounding that mathematicians and logicians have to inform the king of empiricism, physicists.

The King status of physics made the discipline isolated and unaware of the further development of other equally substantial fields such as mathematics and philosophy. For example in "The secret of Gravitational Waves/American Scientists" March-April, 2018, Rothman wrote

> "$\cdots$ a [force] field is a continuously varying plane of action through which disturbances propagate, eliminating the conceptual knot of action at a distance. Today no one doubts the reality of [force] fields, anyone who has sprinkled iron filings on a piece of paper above a bar magnet has perceived a field pretty directly. Back then, the existence of [force] field was less obvious."

May be so it was back then in physics but in logic and philosophy researchers are well aware of the ancient Aristotelian-Augustian modal logic which was further developed by Vatican theologians to try to prove the necessity of God. Modal logic is a study of logic in which we study the logic of necessity and possibility. Now it is a most advanced branch of logic. Logicians clearly understand that what physicists call force field is nothing but a special case of what we call *"counter factual modality"* as we discussed above. Rothman's description of force field as a physical reality clearly proves that he has not sufficient background in the modal logic to understand the modality of force field. We can not physically distribute "force per unit charge (or mass)" all over the continuum space, can we? We can not even directly "see" force exerted at a position in a space. How can we see the force field?

It also is quite clear that just like most other physicists, he does not understand what is continuum and how physical continuum is different from mathematical continuum. Mathematical continuum is *dense*. In between two points, there are infinitely many other points. Moreover, mathematical continuum is closed under limit. It means that for each sequence of elements, there is an element which is the "limit of the sequence". *This structure can never be realized by using what physicists call particles!* As long as one keeps the arrogant attitude of claiming that mathematics is just a language for physics, the person will never understand mathematical continuum correctly. *How can one use a theory which they do not understand as their language? Mathematics is not "calculating". It is for deep understanding which is essential for theoretical physics.* His understanding of physical also is questionable. When we place charges on every point of the field space, this certainly changes the field itself. So, we can not observe the field structure in the way he describes.

Just like the Michelson-Morley experiment, typical "rush to fancy conclusions" culture is pulling the legs of theoretical physics. *The so called em waves and gravitational waves are "transmission" of the "variation of the physical status" of charge or mass at a distance.* As there is no physical medium which propagate such "waves" they are not waves. Philosophically, it is much more desirable to consider just the action (change) at a distance than hypothetical spatial change due to the local change.

It appears that Feynman is one of very few modern physicists who understood this. He rejected force field in his QED. However, what is disappointing is that he failed to recognize that then the so called em waves are not waves. All of this is the consequence of the lack of adequate training in analytical thinking.

As a natural consequence of not understanding what em waves and gravitational waves are correctly, *they appear to have some difficulty in explaining why em waves get "absorbed" as it travels through matters and yet gravitational waves do not.* It is interesting to learn that for them this triviality is a big issue which apparently makes the gravitational wave theory an "extremely challenging" final frontier. Of course this kind of myth is a big boost for the popular science enterprise of theoretical physics (cosmology)

There is no physical medium for gravity, is there? According to the empiricists, they have medium through which electromagnetic force and em waves travel. As empiricists, they are not interested in why the parameters ε and μ for such mediums. They say that these numbers are obtained from experiments one for each medium they used. It was Russell who warned physicists that empiricism is a vicious circle at best. To experimentally verify a theory, we use the theory to be verified to make the apparatus for the experiment.

This is the reason why empiricism failed. It appears that they are not concerned with that what they call the em medium is nothing but a massive sea of charged particles, yes that view material scientists (engineers) developed following the very old Greek philosophy of atomism. This is why the em waves are affected by the "medium". The trick here is that these tiny particles are strong enough in charges to affect the so called em waves.

Certainly the same thing happens to the gravitational forces (gravitons). But due to the G, for tiny mass of particle consisting what we call mass, this effect of gravity upon the gravitational wave (graviton) is negligible. This is why the gravity wave dose not get "absorbed" by the "massive body".

The problem is that as Newton made it clear that if we want to maintain our sanity we reduce mass and charge to point object to avoid this problem. Then we have no head ache of the interference between mass (charge) and gravitational (electromagnetic) waves. *All of this is to say, that in correct dynamics, there is noting but point masses or point charges.* According to the theory, there is no such thing as objects (bodies) through which em waves or gravitational waves travel. Opportunistic and undisciplined mixing up of different categories, typical of the heritage of physics coming from empiricism is sticking its ugly head again.

What has been discussed here is a short but incisive issues and solutions to the early stage gravitational wave theory initiated by Heaviside and further developed by prominent researchers such as Poincarè and Einstein. Though very short, we summed up the issues (and confusions) in the "discussions" by these early pioneers of the field. One of the main problems is that theoretical physicists still have serious difficulty in understanding the difference between physical reality and modality.

8.2 Early works on gravitational waves

Heaviside, using analogy between Newton's law of gravity and Coulomb's law of electromagnetic force suggested the possibility of gravitational waves.

Poincaré, due to the Lorentz transformation in classical electromagnetism theory of Maxwell, which came from the ill fated MM experiment which also is a result of Maxwell's em theory, at least in the context of electromagnetism theory, one can not consider a body which moves faster than c. Due to the "algebraic analogy" between Coulomb's law and Newton's law, Poincaré proposed that the acceleration of a mass should produce gravitational waves as that of a charge produces electromagnetic wave.

One interesting question is what if we close our eyes to the invalidity of STR dynamics of Einstein which considered acceleration in the frame of STR which is limited only kinematics for an obvious reason (the violation of the Principle of Relativity) and studied the gravitational waves in STR dynamics. For some reason, Einstein did not work on this problem. As we will discuss in the next section, he went straight into the gravitational wave theory within GTR. It is plausible that he did not see needs for the gravitational wave theory within the context of STR dynamics as he was developing General Theory of Relativity which is supposed to be a relativistic theory of generalized gravitational field.

8.3 General relativistic gravitational wave theory

Einstein who published General Theory of Relativity in 1915 pointed out that the analogy between Coulomb and Newton breaks down at the dipole, unlike magnetic dipole, there is no gravitational dipole. After all, he explored the idea and came up with the proposal of three different kinds of gravitational waves; namely longitudinal-longitudinal, transverse-longitudinal, and transverse-transverse.

Eddington showed that the first two types of waves of Einstein can propagate at any speed by choosing appropriate coordinate system. He also showed that the third type of Einstein always propagate with speed c regardless of the choice of the coordinate system. Considering the first two types of waves, the physicality of the third case became questionable.

Einstein and Rosen discovered that in full generality of GTR, solutions of gravitational wave equations have singularity. Robertson argued that such singularity comes from the element of the theory which has no relevance to physicality.

In 1956, Pirani pointed out that all of these confusions associated with the Relativistic Gravitational Wave Theory coming from the choice of coordinate systems can be resolved by reformulating the gravitational waves in terms of manifestly observable Riemann curvature tensor.

Next year, Feynman presented a thought experiment which suggests that the gravitational wave carries energy by generating heat using gravitational waves.

8.4 Experimental results

Here is a short history of the experimental results on gravitational wave research.

1. In 1969 and 1970, Weber built the world first gravitational wave detector and detected gravitational waves from the Milky Way's, "Galactic Centre". This result predicted that the life of our galaxy is to be much shorter its inferred age.

2. In 1974, Hulse-Taylor discovered the first binary pulsar. Their pulsar showed a gradual decay of the orbital period which agreed with the momentum and energy loss in the gravitational radiation as predicted by the GTR.

3. Despite the apparent setback of Weber's usage of gravitational wave detector, further development were done on this line. In the 1970's Forward-Weiss made the first notable detector which was followed by the construction of CEO600, LIGO and Virgo.

4. In 2015, LIGO made the first detection of gravitational waves. It was inferred that the signal, dubbed GW150914, coming from the merger of two black holes with masses $36^{+5}_{-4}M\odot$ and $29^{+4}_{-4}M\odot$ resulting in a $62^{+4}_{-4}M\odot$ black hole. They concluded that the detected gravitational wave carried roughly three solar masses or about 5×10^{47}joules.

9 Quantum gravity theory: General review

Beyond the impressive development theoretically and experimentally, there are still some issues to be discussed.

9.1 Gravitational waves of Heaviside-Poincare

In Maxwell's em field theory, the two em wave equations were obtained simultaneously as follows:

$$\frac{\partial}{\partial t}\nabla\times\mathbf{H} = -\frac{1}{c^2}\frac{\partial^2\mathbf{H}}{\partial t^2} = -c\nabla\times\nabla\times\mathbf{E}\frac{\partial^2\mathbf{E}}{\partial t^2} \Longrightarrow \nabla^2\mathbf{E} = \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}$$

Similarly we have

$$\nabla^2 \mathbf{H} = \frac{1}{c^2}\frac{\partial^2 \mathbf{H}}{\partial t^2}.$$

In Maxwell's em theory, the interaction between **E** and **H** came from the current vector **J** as the generalized Ampere Law:

$$\nabla \times \mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{J}.$$

Maxwell's em wave equation is derived under the assumption $\mathbf{J} = 0$. This equation shows that the speed of em wave in vacuum is c. What is puzzling is that the speed of em wave according to this theory is c only when the wave is created in vacuum without current.

Maxwell later showed that accelerating charges generate electromagnetic waves. However such charges create current **J** and then there is no reason to think that the em waves created in this way moves with speed c. So there is something bizarre about Maxwell's theory of em waves.

Remark 27 *Radio engineers say that when we have a closed circuit, it generates an em wave whose frequency is one cycle of the current.*

In case of the gravitational waves, we do not see the "claimed analogy" to em waves. There is only one gravitational field **G**. It seems that the only analogy comes from the acceleration creating waves. But this analogy is weak as we have no such thing as the continuum flow (current) of masses in dynamics. *Moreover the speed c comes not from the nature of the mass but from the ill-fated Lorentz transformation which causes contradictions.*

Remark 28 *If we develop STR version of Gravitational Wave Theory, there are at least two issues to be considered. First, In principle STR rejects gravitation as it is acceleration and thus causes the violation of the Principle of Relativity, which relativists uphold or ignore opportunistically. Second, considering the close tie between STR and Poincré's em field theory, it is well expected that the STR version of Gravitational Wave Theory will encounter similar difficulties to the difficulties Poincaré's theory encountered.*

9.2 Feynman's QED and gravitational wave theory

Considering that Feynman went back to action at a distance electrodynamics to formulate quantum electrodynamics, it is surprising that nobody in gravitational wave theory considered the possibility of action at a distance gravitational wave theory. This might well open up a door to the right theory of gravitational waves and its quantization.

9.3 Gravitational wave theory based on general relativity theory

There are many issues to be discussed. Here we will list some of them. In our paper *"Logical Analysis of Relativity Theory" Abstract presented for "Physics*

Beyond Relativity 2019", Praha, we presented a thorough analysis of the above mentioned idea of Einstein. Here we will present a short version of it.

1. According to the special theory of relativity, relativistic addition of speeds is not $v \oplus v'$ classical addition. So, how can the addition of acceleration be the same as classical addition.

2. This inertial force is also closely bound up with the issue of *"fictitious force"* on a mass inside an orbiting object. Fictitious force means a "force in fiction", not reality. The reason why we have a problem with the fictitious force for an orbiting spaceship is because orbiting spaceship is under centripetal acceleration. It is not an inertial frame. The "creation" of "fictitious force" called centrifugal force does not make the orbiting spaceship becomes an inertial reference frame.

Even more fundamental issue here is considering the spaceship (or train). In the theory of dynamics as Newton made it clear, theoretically there is no such thing as a spaceship (or train). All physical bodies must be reduced to point mass as Newton rightly said. So, there is no such thing as a mass m inside a space ship for this reason.

The most fundamental reason why Newton correctly reduced all moving mass to point masses is simple. It is for purely mathematical and conceptual reason. Newton correctly observed that the best we can do is to consider a physical body as a point object with size of geometric point. Without this assumption, how can we define motion mathematically?

Moreover, for dynamics, we have yet another important reason to reduce a mass to a point mass; it is because force is a vector, a pointed entity. So, the only entity to which we can exert a force is a point object (mass).

There are more issues to be discussed regarding the "gravitational field" Einstein introduced to a space which is under acceleration. The concept of force field, in general, violates the action reaction law, in turn violate the Principle of Relativity. Moreover, the gravitational field Einstein introduced to an accelerating space is a force field which has no source for the gravitational forces spreading all over the space. This is yet another violation of the Third Law in a different sense.

10 Relativistic gravitational waves revisited

After showing that the entire project of GTR carried out by Einstein and Göttingen University is fundamentally flawed, let us go back to the modern day gravitational wave theory which is based upon the concept of gravity which this ill fated GTR offers.

10.1 Singularity problem

Einstein and Rosen discovered that in full generality of GTR, solutions of gravitational wave equations have singularity. Robertson argued that such singularity comes from the element of the theory which has no relevance to physicality. This is an interesting development. At least Robertson agreed that GTR is governed by *physically irrelevant mathematical elements*. Now the question unanswered is if isolating such undesirable elements will yield consistent physically meaningful theory. The record shows that there is no research done to answer this question. *Typical cultural problem of do not miss the bus, get "quick results" to impress the public* diverted researchers from digging into this real problems.

10.2 Source of the confusion: Riemann curvature tensor?

In 1956, Pirani pointed out that all of these confusions associated with the Relativistic Gravitational Wave Theory coming from the choice of coordinate systems can be resolved by reformulating the gravitational waves in terms of manifestly observable Riemann curvature tensor. Again, this observation is highly questionable. Such analysis should follow only after resolving fundamental logical and mathematical inconsistencies of GTR. In an inconsistent theory which is not a theory to begin with, there is no issue of confusion. It is not the matter of tensor. It is the matter of the "momentum-energy" which the tensor is to represent. We strongly wonder what kind of basic logical education these "researchers" were given. It is astounding that the prestigious mathematical institute of Göttingen produced this kind of research. In applied mathematics, mathematics is never the issue. If something goes wrong, it is always the understanding of the issue to which we try to apply mathematics which goes wrong. It is very rare that mathematicians make errors in this way. Mathematics is a self-contained discipline and without any "input" from physicists, mathematicians will articulately figure out if their theory is correct or not.

10.3 Feynman on energy of gravitational waves

In 1957, Feynman presented a thought experiment which suggests that the gravitational wave carries energy by generating heat using gravitational waves. We wonder how he thinks about his own refusal of energy and adoption of momentum to deal with the divergence problem of QED. Is he not yet another example of opportunists?

10.4 Waves which carry momentum?

However, Feynman is not alone at all. Virtually all theoretical physicists became too specialized and look at things only through narrow windows of their hopelessly specialized highly questionable research. For example according to https://en.wikipedia.org/wiki/Gravitational_wave", water, waves, sound waves and electromagnetic waves are able to carry energy, momentum and angular momentum and by doing so, they carry those away from the source."

Anybody who studied wave mechanics correctly knows that no water wave which moves through water will carry momentum as there is no momentum in such waves. No mass moves in the direction of the motion of water waves. It is the local vibration of the medium which moves to the direction of the waves. No earth quake will move water as tsunami from the epicentre to many thousands of km away with the speed of 500km/h. The problem with the theoretical physics culture is that they become so "superior" and "removed" from the reality that they do not understand what the secondary school students will readily understand anymore. As we keep stressing, there is no such thing as em waves as em field is not physical reality. It is a counter factual modality. More fundamentally energy is not a legitimate physical concept as we discussed above. The work needed to accelerate from $m0$ to mv is not necessarily $mv^2/2$. It depends upon how we accelerate. What is more disappointing is that when we discuss these basic issues, the usual responses we get are typical arrogant and rather off the point "responses".

The idea of waves carrying momentum came in as the result of the false theory of relativity. Einstein's ill fated $e = mc^2$. This lead to the wrong equation of momentum-energy which related momentum and energy. This lead to the wrong conclusion that as waves carry energy, they carry momentum. This is not only mathematically wrong but also conceptually false. Energy is a counterfactual modality and momentum is a physical reality. They are of different category.

It is disconcerting that the political dominance of relativity which lasted nearly a century eroded theoretical physics to the extent where theoretical physicist can not separate classical wave mechanics and relativistic wave theory. The culture of suppression of threatening facts put the physicists community in the dark, hampering the future development of physics.

10.5 Observable Riemann curvature tensor?

Einstein and Rosen discovered that in full generality of GTR, solutions of gravitational wave equations have singularity. Robertson argued that such singularity comes from the element of the theory which has no relevance to physicality. In 1956, Pirani pointed out that all of these confusions associated with the relativistic gravitational waves are coming from the choice of coordinate systems and can be resolved by reformulating the gravitational waves in terms of manifestly observable Riemann curvature tensor.

The problem here is not technical nature as Pirani thought. The question is what did he mean by "observable". Is it observable in the local Minkowski space or in the absolute space. If mathematicians are asked, they will certainly say that it must be in the absolute frame as the absolute space curvature is defined by the energy-momentum tensor.

So, far so fine, but what about actual measurement? We have our own local Minkowski space according to Einstein and contingent upon the grand assumption that the relativity inside this local space is consistent and valid, we can legitimize our observation in this local space. There are two issues here. First, Minkowski local space time theory is inconsistent. This finishes the entire argument here. Second, the Riemann tensor defines the curvature of the absolute space. How local observation of local space can lead us to the global curvature of the absolute space.

10.6 Red shifting

It has been argued and accepted by the mainstream physics community that the electromagnetic wave shifts towards the red due to the relative velocities of the source and observer. The argument goes as follows: Assume we emit a light beam upward from the floor to the ceiling. Due to the downward acceleration of the room, by the time the light reaches the ceiling the ceiling is moving faster than the source on the floor was when light left it. In other words, the receiver at the ceiling is approaching the source (to be precise where it was when light left). Therefore we should expect the blue shift due to the Doppler effect. Therefore the observer in the room will notice the blue shift. This will make the observer notice the downward acceleration. This contradicts the Equivalence Principle which says that the free falling body will not notice its free falling. So, there must be a red shift due to the light moving upwards against the gravitational to compensate this blue shift. This is how Einstein obtained the red shift.

This argument is flawed. It is not the observer in the room who sees the ceiling falling towards where the floor used to be. It is an outside observer who will see that the ceiling is falling towards where the floor used to be. Einstein failed to understand that the inside observer is also subjected by the same acceleration due to the gravity. After all, all of this confusion is simply due to the simple fact that Einstein's argument for the Equivalence Principle is fundamentally flawed.

There is an even more elementary flaw in this argument of Einstein. Einstein's failed argument applies only for the accelerating velocity of the source and observer.

Notwithstanding, gravitational wave research community endorses the view that like em waves, gravitational waves exhibit shifting of the wave length due to the relative speed of the source and observer. To begin with this claim is false because the analogy must apply for the case where the source and receiver are not relative inertial motion but relative accelerating motion.

This issue shows the worst of the troubled relativity theory. Relativity theory has different faces depending upon the nature of the relative speed, either constant speed or time varying speed. It is shocking that no professional relativists paid enough attention to this subtle but important issue.

11 Quantum gravity theory

Among enthusiasts, there is a big urge to build "quantum gravity theory" which is analogous to the "quantum electrodynamics". They have some difficulty in dealing with the divergent terms. For some reason, they can not find a good way to deal with this.

11.1 Quantization of gravity waves

Considering the fact that in QED, we started with the quantization of em waves as photons which are "rest mass" zero particle which "keeps moving with constant speed c", may be the first thing to do in this direction here is to obtain the "quanta of gravitational wave".

However, we have some problem with Einstein's photon We wonder how is it possible that a particle which never stops, which always move with speed c has "rest mass". Having rest mass which is 0 and having no rest mass at all are entirely different category. Let us make this argument more concrete. Consider $x^2+1=0$. This equation has no real solution. *Does this mean $x=0$ in a theory where we do not consider imaginary numbers?*

As discussed in *[2] Relativistic theory of photon*, Einstein thought that the rest mass of photon is 0 as he thought that 0/0 is "any number". But this immediately leads to contradictions.

After all, is it still a good idea to follow the step of what QED researchers did and bang out quantum version of gravity waves? As a typical cultural problem of theoretical physics, instead of rectifying wrong ideas and wrong results, researchers tend to push the old questionable ideas to push their frontier further forward. This following the trend continues till the next "revolution" of sensational scale appears. The entire history of the 20th century theoretical physics was this process of pushing wrong ideas and wrong method which created inconsistent theories to expand the kingdom of wrong thinking. We have argued with solid reasoning that QED is not the most successful theory of physics. It is one of the most questionable of physics in history. This is not surprising at all if we consider the dependence of this theory upon the completely false theory of relativity. What is lacking in contemporary theoretical physics is discernment.

The reason why QED appeared to be the most successful theory is mostly because it dealt with extremely small particles moving with incredibly fast speed. This makes it impossible to do what we are used to do as "experiments" impossible. It is clear that almost all experiments were measured applying theoretical formulas to trajectories produced "which are not even supposed to be there" according to the theory. Certainly the formulas used to measure are all verified It is what logicians call vicious circle. To make the matter even worse, probability theorists will remind quantum mechanists that there is no such thing as experimental verification or refutation for QM as it is a theory of probabilistic prediction and the relative frequency converges only at the limit.

It appears that physics community is still trapped in the mind set of good old industrial revolution age in their mind set and what we are dealing with in contemporary physics is way more advanced than the industrial revolution time physics. Physicists ought to open their eyes to this new situation and figure out the correct and new way to deal with today's issues in physics.

So, instead of wasting resources in pushing this wrong project based upon wrong ideas, we should go back to the source of problems and correct them. The first thing we must do is to understand the total invalidity of relativity theory and do something about it. This obviously leads to the correction of QM and eventually the theory of gravitation.

11.2 Renormalization problem

As we discussed in *6.5.4 Tomonaga-Schwinger renormalization,* in QED, the reason why they managed to find solutions to the problem of reorganization is because they treated the issue as a pure mathematical issue as Tomonaga did. He showed that for the problems which appear in QED, the diverging terms are always linear or logarithmic. So, he managed to show, for these problems, how to remove the diverging term problems. As Dyson showed, Feynman's solution to this problem, which he used Kramer's cut off, is a special case of the general solution of Tomonaga.

In case of quantum gravity theory, apparently this is not the case. But is this the weakness of quantum gravity theory? We do not know. We do not think that the solution Tomonaga presented is intrinsic. It was just a matter of luck that Tomonaga found that the diverging problems in QED are caused by the linear or logarithmic terms.

In the end the renormalization issue for QED is closely bound up with the good old issue of the electromagnetic mass which appeared in the classical em theory as closely studied by Thomson and Poincaré. *So, it had some ontological connection to reality.* We do not see this relation in quantum gravity theory. The difference here is that in case of QED, it was such that the em force field added extra mass to the charges. In case of gravitational field theory, there is only one force, gravity.

All of this is to say that contrary to the development of the theory of renormalization in QED, there is no ontological explanation of the problem of renormalization in the quantum gravity theory. It is a pure "mathematical" theory which has little relevance to physics.

The historic reality is such that the well-understood fundamental connection between classical em theory and QM was severed by the ultra creative theory of relativity by Einstein, as more and more theoretical physicists are recognizing, and this "grand mathematical abstraction" made theoretical physics drifted away from physical reality to the wonder land of popular science. From mathematical point of view it is highly questionable if mathematics used in modern physics such as relativity theory was really understood by those who used it. For example it appears that basically no specialists on GTR understands a basic mathematical fact that the manifold can exist only within the context of Euclidean geometric space. This lack of understanding made relativists attack Kant for his philosophy of the space. These trend following thinkers did not understand that without Kantian philosophy of space, there is no curved space. Among modern theoretical physicists, Feynman seemed to have been one of the very few who rightly did not take relativity theory seriously. He also is known for rightly rejecting the highly controversial force field concept in theoretical physics for a very good reason. It is unfortunate that he has not been appreciated for really important things he did. As a popular science champion figure, he has been "known to the public image as a genius" created by the business interest of popular science. All of this is consistent with the disturbing fact that Einstein whose mathematics is mostly wrong has been known to be a mathematical genius. In pure mathematics, we have no such thing as geniuses. We call Gauss a greatest mathematician ever lived.

11.3 Gravitons?

Despite all of these difficulties and negative perspectives which are emanating from the mentioned conceptual and mathematical incoherence, dedicated Quantum Gravity researchers still hold a high hope in their future. They consider the project of building successful theory of quantum gravity to be a *harmonization* (*unification*) of gravitational theory and quantum mechanics: one for the understanding of large scale physics and the other for micro scale physics.

11.3.1 Empirical issues

From empirical point of view, this is highly questionable. Astronomy is way too large for us man kind to do experiment in the way we do experiment for our size world. All we can do is to observe the far way starts relying upon em waves. As we know well, basically we know nothing about em waves and lights except that they are not physical reality. They are modal waves which travel through the modal fantasy of the em fields. Moreover, the only theoretical tool which gives us power to conclude something about distant stars is the so called Doppler effect. Despite so much trust we have on this effect, the logical reality is that we have no understanding of the connection between the frequency shift and the energy conservation. Frequency shift means the shift of energy and there seems to be no clear explanation of this observed energy shift and the total energy conservation. So, we are not sure if our calculating the speed of a distant star through the Doppler effect is trustworthy. On the top of it, once we get this speed correct, then we have to translate it to the relativistic one and clearly there is a vicious circularity in this argument. The problem of v in the gamma factor being classical speed is sticking its ugly head up.

Quantum particle world is way too small for our size world to do experiment. This is why in QM we ended up with the probabilistic theory with uncertainty. This killed entire quantum physics because the uncertainty principle asserts that we can never ever see trajectories. We wonder what then are these lines which appear in the particle detection chamber. A typical "response" from the mainstream quantum mechanists on this fatal criticism was totally astounding. *They said it was not the problem of QM but it was the problem of Copenhagen interpretation!* Jolly good, the ignorance and arrogance exhibited by this is totally consistent with that exhibited by the typical statement from the mathematically challenged physicists that *mathematics is just a language for physics.*

The bad news is that neither astronomical scale physics nor micro-scale physics can be handled by the "traditional empirical method" anymore.

What is interesting is that in the so called quantum computing, more and more researchers are distancing themselves from QM thinking that this is an interesting parallel computing at the level of bits which should be realized out of the QM.

So, we wonder where the minds of those quantum gravity researchers are when they dream about harmonizing these two extreme world which we have little control.

11.3.2 Theoretical issues

Putting aside this warning from empirical point of view, we wonder if it makes any sense to dream about "meaningfully unifying" quantum physics and general relativity theory both of which are mathematically and logically inconsistent. Logic clearly tells us that an inconsistent theory can prove anything. It is called deductive explosion in logic and mathematics. This is why we reject inconsistent theories. All of this simply means that already quantum theory and general relativity are "harmonized and unified". The only draw back here is that the "harmonized theory" is meaningless and so totally useless.

Putting this sweeping but true statement aside, let us discuss some attempts made on the line of unifying quantum mechanics and general relativity to produce quantum gravity theory.

Problem of time: One of the largest issues here is that of time. In quantum theory, time is a meta concept which defines physical phenomena but is not affected by physical phenomena. In general relativity theory, time is a variable which is affected by other elements of the system. Logically speaking, the time in general relativity theory is invalid due to the vicious circularity or self-refutation. It is unfortunate that relativists do not understand this hierarchical issue. In serious mathematical science, we do not deal with over creativity like this as it leads to contradiction. Science and fine arts are entirely different category. We scientists do not want this kind of undisciplined creativity. Mathematical science is not an indulgence in creativity. It also is the case for fine arts where more liberal creativity manifests and play key role. They do not need strictly disciplined creativity as in mathematical sciences. Anyhow, these two theories are inconsistent in different ways and so, in the end this difference does not matter.

Problem of point particle: Despite some sign of success at low energy, the project of quantum gravity hit a wall at high energy where the renormalization, removing the divergence due to the point nature of the particle, became impossible. Putting aside the validity of the "quantization" in the sense of quantum field theory, this difficulty lead some theoretical physicists to a "mathematical; solution" in which we avoid the point particles all together and moved to the physics of *strings*. This theory is capable of modelling difference in particles such as charges and gravitons involving "10 dimensions". This *super string theory* is not relativistic in the sense of based upon "curved spacetime" of Einstein.

M-theory: The effort to make the above mentioned super string theory general relativistic created an M-theory which is an eleven dimensional theory. Despite claimed "success' this theory also succumbs to the inconsistency problem which comes from that of the General Theory of Relativity.

In the end, the tragic conclusion is that all of these great efforts turned out to be failure just because both STR and GTR are inconsistent, in addition to the inconsistency of QFT such as QED which originates from the inconsistency of STR. This is a clear warning to theoretical physics that the logical consistency of a physical theory is a most important issue not to be put aside. By denying it, as it has been the case for nearly a century, the problem will not go away. The devastation will keep accumulating to the catastrophic end. To be more direct, finding political solutions to this kind of problems will never resolve the problem. Now theoretical physics is at cross road as Dingle warned 50 years ago.

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