The Ghost of Modality in Quantum Physics

Abstract for Invited Presentation for "Physics Beyond Relativity 2019"

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1 Mathematical continuum v.s. physical continuum

1.1 Mathematical continuum

The term continuum has been used casually in theoretical physics, causing some alarming situation. We will briefly discuss what continuum really means mathematically so that physics will not step into some fundamental conceptual errors in considering continuum structure.

A function f from set A to a set B which is one-to-one and onto is called a *"bijection"*. A set X is "countable" if it is a finite set or there is a bijection from the set N of all natural numbers to X. In more conventional way, we can say that a set X is countable if it can be expressed as

$$X = \{x_0, x_1, x_2, ...\} = \{x_i : i \in N\}$$

Example 1 *1. The set E of all even numbers is countable, as the function $f : N \to E$ such that $f(n) = 2n$ is a bijection. Similarly, the set O of all odd numbers is countable. 2. The set of all rational numbers is countable. To show this we first recall that all rational numbers can be expressed as n/m, where n*

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and m are natural numbers and $m \neq 0$. Now we can list all rational numbers as

$$
\begin{array}{ccccccc}
\frac{1}{1} & & \frac{1}{2} & \rightarrow & \frac{1}{3} & & \\
\downarrow & \nearrow & & \swarrow & & \nearrow & \\
\frac{2}{1} & & \frac{2}{2} & & \frac{2}{3} & & . \\
& \swarrow & & \nearrow & & \swarrow & . \\
\frac{3}{1} & & \frac{3}{2} & & \frac{3}{3} & & . \\
\downarrow & \nearrow & & \swarrow & & \nearrow & . \\
& & . & . & . & . &
\end{array}
$$

Cantor "hypothetically" listed all elements of an open interval $(0, 1)$ as follows:

$$
\begin{array}{l}
0.d_{11}d_{12}d_{13}, ... \\
0.d_{21}d_{22}d_{23} .... \\
....
\end{array}
$$

He created a new real number as $x = 0.d_1d_2d_3, ...$ such that $d_1 \neq d_{11}$, $d_2 \neq d_{22}$, $d_3 \neq d_{33}$, ... Clearly x is in $(0, 1)$. But it cannot appear in the listing above at the pain of contradiction. So he rightly concluded that the set R of all real numbers is not countable.

Indeed, as we discussed above, we can enumerate all rational numbers but we can *not* enumerate all irrational numbers. Indeed, we can show that almost all real numbers are irrational numbers using Weierstrass function which is defined over the interval $(0, 1)$ as

$$w(x) = \textbf{if } x \text{ is rational } \textbf{then } 0 \textbf{ else } 1$$

The Lebesgue integral of this function over $(0, 1)$ is 1.

Any interval (a, b) of real numbers is called *"continuum"*. Clearly there are way more points in the continuum than in countable sets, as we discussed above.

1.2 Physical continuum

It has been assumed as an "empirical common sense" that there are at most countably many (more likely finitely many) particles in the universe. This assumption comes into a conflict with the mathematical reality. We have been told that the wave frequency of electromagnetic waves in theory can be any real number in the open interval $(0, \infty)$ of real numbers. This means that there are "continuumly many frequencies" of electromagnetic waves. This implies, according to the relativistic theory of electromagnetic waves as per Einstein, that there are continuumly many particles called photons in this universe. The record shows that Planck disagreed with the idea of considering $h\nu$ a particle called photon. For him this was just a mathematical convention. This is a very good example of a serious discrepancy between the concept of continuum in mathematics and that in theoretical physics.

Another manifestation of the discrepancy between the continuum for physics and that for mathematics can be seen in the fluid mechanics. In this theory, they consider a *"force applied to a unit area"*. They call it "pressure". In Newton dynamics, all physical bodies are reduced to point bodies and force as a vector is applied to a point body not to a body with geometric dimension. So, purely theoretically, *there is no such thing as applying force to a non-point body such as unit area.* More fundamentally, a unit surface is made of continuumly many geometric points and this leads to the assumption that just a unit area has uncountably infinite number of particles. This is a serious violation of the basic assumption on our universe where we assume that there are at most countably infinite particles.

Exactly the same thing happens in wave mechanics. For example, when we consider the so-called string waves, there are continuumly many geometric points in a string and each of them as a particle is supposed to be subjected to force. This is in conflict with the most fundamental assumption of dynamics that there is at most countably infinite number of atoms.

This is to say that the theory of particle dynamics and that of continuum dynamics are entirely different things. The very concept of motion and force in the particle dynamics and that in continuum dynamics are entirely different.

The calculus physicists use is based upon the reasoning which contradicts the empirical expectation on the "number of particles".

1.3 Infinitesimals in physics and in mathematics?

Mathematically speaking, this problem is directly linked to the way physicists use calculus. "dx" in calculus means an "infinitesimal" which does not mean a very small real number as physicists think. *An infinitesimal means a positive "number" which is smaller than any positive real number!.* As Cantor pointed out, the so-called "calculus" which physicists use as a "language" is based upon this apparently paradoxical concept. Newton, the founder of calculus was reluctant to use infinitesimals. Leibniz endorsed infinitesimals, though just like everybody else, he was not sure what it meant. One thing for sure is that whatever infinitesimals are, they are not real numbers.

Cantor, the founder of set theory, *openly* rejected this concept of infinitesimals as a *paradoxical concept.* Mathematical analysts (researchers of advanced calculus) avoided this mysterious concept altogether and used the *topological concept of limit* to develop "precise calculus" which they called "mathematical analysis". As mathematics this is perfect but we lost direct connection with physics. Later, Newton's calculus which used naive limit concept was verified by the topological approach. But limit is not an ontological process.

It was Leibniz who used the naive infinitesimals to develop calculus, which, on the surface, is equivalent to Newton's calculus. Mathematicians avoided this difficult approach. It was the mathematical logician Abraham Robinson who in 1960 developed a correct theory of infinitesimals using the construction of "ultra power" which was developed by himself for a branch of mathematical logic called model theory. This is the only correct theory of calculus which uses infinitesimals that is available now.

Physicists have a naive version of infinitesimals. For them dx simply means a "very very small positive real number". Putting the connection between physicist's calculus and topological calculus aside, there is no connection between Robinson's infinitesimal calculus and physicist's version of infinitesimal calculus. This is to say that the "calculus" physicists use is not! Indeed, it is clear that a very small positive real number and a positive number that is smaller than all positive real numbers are entirely different things.

When we discussed this problem with theoretical physicists they said they do not care. When we contacted mathematicians, they replied that they know the problem but, for political reasons, they do not want to get involved. Newton was a theologian, a mathematician and a physicist at the same time. Did he ignored himself? Maybe his Orthodox theology helped him to maintain his own integrity.

2 Wave-particle duality of de Broglie and Schrödinger's equation

2.1 De Broglie's relativistic wave-particle duality

De Broglie obtained the following relativistic transformation of a plane wave for a plane wave which is invariant under the Lorentz transformation (we call it a "relativistic wave"):

$$k'_x = \frac{1}{\sqrt{1-(v/c)^2}}\left(k_x - v\frac{\omega}{c^2}\right), \quad k'_y = k_y, \quad k'_z = k_z, \quad \omega' = \frac{(\omega - \nu k_x)}{\sqrt{1-(v/c)^2}}$$

where $\mathbf{k} = (k_x, k_y, k_z)$ is the wave vector and ω is the frequency. We denote the wave number $|\mathbf{k}|$ by k. So, $k = |\mathbf{k}|$. This restriction to "relativistic waves" is because otherwise the wave phase $\mathbf{k}\cdot\mathbf{r} - \omega t$ will not be invariant under the Lorentz transformation. Using the analogy between this and the momentum-energy transformation of relativity dynamics of Einstein,

$$p'_x = \frac{1}{\sqrt{1-(v/c)^2}}\left(p_x - v\frac{E}{c^2}\right), \quad p'_y = p_y, \quad p'_z = p_z, \quad E' = \frac{(E - \nu p_x)}{\sqrt{1-(v/c)^2}}$$

where $\mathbf{p} = (p_x, p_y, p_z)$ is the momentum vector and E is the energy, de Broglie "proposed" the following association between a particle and a wave (called matter wave):

$$\mathbf{p} = \hbar\mathbf{k} \quad E = \hbar\omega$$

where $\hbar$ is a constant. We call this *"de Broglie (wave-particle duality) relation"*. This is how the wave-particle duality of quantum mechanics was introduced.

Remark 2 *This was in analogy to Einstein's wave-particle duality between electromagnetic waves and photons*

$$E = h\nu = pc \qquad p = h/\lambda$$

which turned out to be invalid at the pain of mathematical contradiction.

Lorentz transformation is defined in terms of the constant c, which is the speed of electromagnetic waves in vacuum. So, there is no ontological reason to think that this transformation will conserve wave functions which are not electromagnetic wave equations of Maxwell.

In the foregoing, we discussed the wave-particle duality of light wave and photon as per Einstein and that of matter wave and relativistic particles as per de Broglie. Despite this promising "analogy", there is a fundamental difference between these two dualities. Light-photon duality of Einstein does not lead to the so-called "uncertainty principle". Indeed, if position-momentum uncertainty holds for photon-light, we will never observe photon (light) as the speed of photon is constant c. Contrary to that, de Broglie's particle-wave duality, as embedded in von Neumann's formalism (more essentially Schrödinger's formalism as von Neumann's formalism is a variation of Schrödinger's), leads to uncertainty principle. So, de Broglie's wave-particle duality is not quite a generalization of Einstein-Planck's wave-particle duality. In other words, *the claim that photons are quantum particles is a myth.*

2.2 Schrödinger's quantization of Hamiltonian dynamics

2.2.1 Schrödinger's wave equation

Schrödinger used Hamilton's energy dynamics for the particle theory and applied de Broglie's pilot wave theory to produce a wave-particle duality which looks after the energy issue of de Broglie's relation.

All waves propagated along the x-axis obey the following wave equation

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{\omega^2}\frac{\partial^2 \Psi}{\partial t^2}$$

where $\Psi(x,t)$ is the wave function and ω is the wave speed.

Here, we consider the wave function Ψ whose square yields the probability of locating a particle at any point in the space. We consider only systems whose total energy E is constant and whose particles move along the $x-$axis and are bound in space. Then the frequency associated via "de Broglie relation", which is totally hypothetical and relativistic, with the bound particle is also constant, and we can take the wave function $\Psi(x,t)$ to be $\Psi(x,t) = \psi(x)f(t)$. As the frequency is assumed to be precisely defined, $f(t) = \cos 2\pi\nu t$. So, we have

$$\frac{\partial^2 \psi}{\partial x^2} = -\left(\frac{2\pi}{\lambda}\right)^2 \psi = -\left(\frac{p}{h}\right)^2 \psi$$

where the wave length is $\lambda = \omega/\nu$ and the momentum of the particle is $p = h/\lambda$.

We take the particle of mass m to be interacting with surroundings through a potential-energy function $V(x)$. The total energy of the system is given by

$$E = E_k + V = p^2/2m + V$$

where E_k is the kinetic energy of the particle. Then we have $p^2 = 2m(E - V)$. This leads us to

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = E = i\hbar\frac{\partial\psi}{\partial t}.$$

This equation, is called the Schrödinger wave equation. *This equation is a "relativistic" wave equation for Hamilton's particle equation.* In reality, as the energy equation involves "non-relativistic mass" m it is not invariant under the Lorentz transformation. This does not mean that quantum mechanics is a non-relativistic theory. The derivation of Schrödinger's wave equation involved de Broglie relation, which is relativistic. Here we have ended up with a situation where we have an important equation which is "half relativistic and half classical".

Nevertheless, Schrödinger knew this problem and he tried, unsuccessfully, to make his wave equation relativistic. This problem was "resolved" by Gordon-Klein-Dirac in the development of the quantum electrodynamics.

2.2.2 Uncertainty Principle

Schrödinger tried to reverse the process of converting particle equation to wave equation to complete the desired wave-particle duality. Here he encountered a fatal difficulty. One of the most fundamental issues in measuring waves is that there is an inherent uncertainty (ambiguity) in measuring the waves, namely that measurement of waves must involve the measurement of counting the number of crests passing at one fixed point (observing point) in the path of the wave. In general, we have a difficulty in deciding what to do with the situation where the point is in-between two crests passing at the moment of measurement. This problem always manifests itself when we try to measure the material waves. This problem is called the "fundamental uncertainty problem".

$$\Delta\nu\Delta t \simeq 1.$$

Since $E = h\nu$, the uncertainty above brings in the following uncertainty: $\Delta E\Delta t \simeq h$.Similarly, upon the measurement of momentum/location, we have the following uncertainty: $\Delta p\Delta x \simeq h$.

Unfortunately, this problem does not disappear in a more advanced (mathematically) formulation of quantum mechanics due to von Neumann. This is because his Hilbert space formulation still is based upon Schrödinger's wave equation, which is based upon the wave-particle duality of de Broglie.

2.2.3 Problems with the uncertainty principle

The ultimate destiny of this attempt to cross the line between continuum and discrete is the *"empirical refutation" of the entire theory of quantum mechanics*. Uncertainty of quantum mechanics predicts that once we localize a particle, the momentum becomes impossible to be predicted, resulting in the impossibility of observing trajectories of particles. In all particle collider experiments, particles are producing trajectories and physicists use these trajectories to study the particle world. In this way, quantum mechanics took the empiricism away from physics. But in many senses, this seems to make more sense than otherwise. Though it has been claimed that quantum theory is the most empirically verified theory in the history of physics, it appears that this theory also is the most empirically refuted theory in the history of physics.

2.2.4 Why the empirically most verified theory?

In contrast to what we discussed above, quantum electrodynamics is known as "the most empirically verified theory" of physics in history. This in turn supports the "ultimate empirical validity" of the special theory of relativity. What is most astonishing is that, despite all of these claimed experimental verifications, this theory came from trajectories produced in the particle detection chamber.

We will also question the role of relativity theory in interpreting the experimental results coming from the trajectories produced in particle detection apparatuses. The trajectories are caused by the (charged) particles colliding with the gas molecules inside the chambers. This means that relativity theory does not apply in the chamber. Despite this, all theoretical analysis of the trajectories is based upon relativistic formulas. To make matters even more confusing, there seems to be no frame moving in this situation. Our frame is the frame of the particle detecting chamber and the particles are moving in it. So, why would we need relativistic theory here.

What is becoming rather clear is that in physics, fancy theories (with mathematical and conceptual incoherence) are developed and experimentalists apply these fancy formulas to their experiments rather mechanically without due analysis of the experiments to determine if the formulas are applicable to their situation.

Above all, it may be worthwhile to figure out why predictions of this theory are "experimentally verified" with a record high success rate? It is a very interesting question. To investigate this issue, it may be helpful to pay attention to what Bertrand Russell said on empiricism in a general setting. He said,

1. Empirical verification of a theory is a vicious circle as the experiment to verify the theory uses the theory to be verified.

2. Empirical refutation of a theory is a contradiction as the experiment to refute the theory uses the theory to be refuted.

It was unfortunate that this did not attract attention from physics community.

Going back to the original question: there is a convincing answer. In particle physics experiments, since particles are way too small to be picked up and measured as we do in classical physics, we rely upon the trajectories particles leave and apply formulas of quantum electrodynamics to measure. So, measurements are done as calculations. *Then naturally the formulas used are automatically verified.* It is a vicious circle.

Example 3 *Here is an example which reveals this problem: we will examine a typical argument of particle physics in which they experimentally find an unknown particle mass in terms of known masses and measured energies and momenta. The experimental data is the trajectories that appeared in a bubble chamber in violation the uncertainty principle, which theoretically refutes any chance of observing trajectories, as we have been discussing throughout. In this experiment a beam of very high energy negatively charged k^{-1} mesons enters the bubble chamber. The following two possible particle reactions take place*

$$k^- + p^+ \to \Lambda^0 + \pi^0 \qquad K^- + n \to \Lambda^0 + \pi^-$$

where p^+, n are proton and neutron, respectively, and Λ^0, π^0 and π^- are an uncharged hyperon, an uncharged pion and a negative pion, respectively. As Λ^0, π^0 are neutral electronically, we do not observe the tracks of them. However, the following particle interaction follows: As the resultants of this reaction are both charged, we can observe the trajectories of them (p^+ and π^+).

$$\Lambda^0 \to p^+ + \pi^+.$$

At a point A, a k^- track disappears due to the equation for k^-. A Λ^0 is produced that travels to point B where it decays into p^+ and π^+. The direction of π^+ relative to AB is θ_π and that of p^+ is θ_p. We will apply relativistic laws of conservation of energy and momentum to this experimental data of the decay of Λ^0 at B for the measuring of the rest mass $m_{0\Lambda}$ of Λ^0. We assume that the speed of Λ^0 is v_Λ and its energy is

$$E_\Lambda = m_{0\Lambda}/\sqrt{1 - v_\Lambda^2/c^2}.$$

Then we have

$$E_\Lambda = m_{0\Lambda}/\sqrt{1 - v_\Lambda^2/c^2} = E_\pi + E_p \qquad (1)$$

where E_π and E_p are relativistic energy of pion and proton. By the conservation of relativistic momentum, we have

$$\begin{aligned} p_\Lambda &= p_x \cos\theta_\pi + p_p \cos\theta_p \qquad (2) \\ 0 &= p_x \sin\theta_\pi - p_p \sin\theta_p \qquad (3). \end{aligned}$$

Dividing (2) by (1) and $p = mv = E_v/c^2$, we have

$$v_\Lambda/c = p_\Lambda c/E_\Lambda = (p_x c\cos\theta_x + p_p c\cos\theta_p)/E_\pi + E_p.$$

All of this leads to

$$m_{0\Lambda}c^2 = (E_\pi + E_p)\sqrt{1 - v_\Lambda^2/c^2}.$$

As

$$p = \sqrt{E^2 - m_0^2c^4}/c = \sqrt{T^2 + 2m_0c^2T}/c$$

where T is the kinetic energy, for the experiment with $T_p = 44MeV$ and $T_\pi = 60MeV$, we have

$$p_p = \frac{291MeV}{c}.$$

The rest energy of π^- is $139.6Mev$, which leads to

$$p_\pi = \frac{143MeV}{c}.$$

Moreover, many other settings of the experiment lead to quite close calculated results. From this particle physics researchers claim that the relativity theory is experimentally verified. *What is happening here is what logicians call vicious circle. If we interpret some experiments using calculation of "a" theory, naturally "such theory" is verified by the experiments we do. This is a near fatal logical issue for empiricism. This is why relativity theory has been experimentally verified most often in the history of physics despite that this theory is completely inconsistent. The problem with this is that they did not measure the rest mass $m_{0\Lambda}$ of Λ^0 directly by measurement. This is a clear case where traditional macroscopic empiricism does not connect to modern microscopic theories.*