

Critique of the Einstein clock variable

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Abstract: This paper presents a fundamental and far-reaching criticism of the special theory of relativity. It aims to rescue the precious variable t from the artificial interpretation that Einstein gave it. Einstein defined a certain variable and called it time. I show that calling this variable time is extremely misleading. Instead I call it the Einstein clock variable, and I discuss the role of clocks in the scientific project of describing nature without using the word “time.” Einstein defines his clock variable as a fourth variable belonging to an inertial coordinate system. Each inertial coordinate system has its own instance of the Einstein clock variable, just as it has its own instances of three distance variables. A coordinate system’s instance of the Einstein clock variable is defined by a spatial array of indefinitely many clocks, all at rest relative to the object that anchors the coordinate system, and all coordinated with one another using a method that Einstein specifies. The key finding of this paper is that Einstein’s method of coordinating clocks results in a variable that has a nonzero spatial gradient. The value of the variable varies systematically with location, much as atmospheric pressure varies with altitude or the readings on clocks in airports around the world vary with time zone. All the sensational claims of the special theory of relativity, including the relativity of simultaneity, time dilation, length contraction, and the merging of space and time, use familiar words as redefined by Einstein to describe aspects of the relationship between two differently sloping instances of the Einstein clock variable. There is nothing in nature corresponding to any of these claims. The spatially sloping clock variable on which they are based is a thought-warping artifice that has no business in descriptions of nature. If physics is to be a *natural* science, this variable has no business in physics. © 2019 *Physics Essays Publication*.
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Résumé: Cet article présente une critique fondamentale et profonde de la théorie de la relativité restreinte. Il vise à sauver la précieuse variable t de l’interprétation artificielle donnée par Einstein. Einstein a défini une certaine variable et l’a appelée ‘temps’. Je montre que l’appel de cette variable ‘temps’ est extrêmement trompant. Au contraire, je l’appelle la variable d’horloge d’Einstein et je discute du rôle des horloges dans le projet scientifique consistant à décrire la nature sans utiliser le mot ‘temps’. Einstein définit sa variable d’horloge comme une quatrième variable appartenant à un système de coordonnées inertiel. Chaque système de coordonnées inertiel a sa propre occurrence de la variable d’horloge d’Einstein, tout comme il possède ses propres occurrences de trois variables de distance. Une occurrence de la variable d’horloge d’Einstein est définie par un ensemble spatial composé d’un nombre indéfini d’horloges, toutes au repos par rapport à l’objet qui ancre le système de coordonnées, et toutes coordonnées à l’aide d’une méthode spécifiée par Einstein. La principale conclusion de cet article est que la méthode de coordination des horloges d’Einstein donne une variable qui a un gradient spatial non nul. La valeur de la variable varie systématiquement avec la position, de la même manière que la pression atmosphérique varie avec l’altitude ou que les lectures des horloges dans les aéroports du monde varient avec le fuseau horaire. Toutes les affirmations sensationnelles de la théorie de la relativité restreinte, y compris la relativité de la simultanéité, la dilatation du temps, la contraction de la longueur et la confluence de l’espace et du temps, utilisent des mots familiers tels que redéfinis par Einstein pour décrire les aspects de la relation entre deux occurrences diverses de la variable oblique d’horloge d’Einstein. Rien dans la nature ne correspond à aucune de ces situations. La variable d’horloge oblique spatiale sur laquelle ils sont basés est un artifice de distorsion de la pensée qui n’a pas un rôle dans les descriptions de la nature. Si la physique doit être une science *naturelle*, cette variable n’a aucun rôle en physique.

Key words: Time; Clock; Coordinate System; Nature; Velocity of Light; Lorentz Transformation; Maxwell’s Equations; Special Theory of Relativity; Einstein.

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I. OBJECT-ANCHORED COORDINATE SYSTEMS

Physicists make extensive use of the idea of an object-anchored Cartesian coordinate system. This is a Cartesian coordinate system whose origin is attached, in the imagination of those who use it, to a particular physical object. Three mutually perpendicular axes are imagined to intersect at the point of attachment and to extend outward through the surrounding space. The numbers along these axes have units of length.

Like Mary and her lamb, the anchoring object and the coordinate system it anchors travel together. If two objects are in motion relative to each other and each of them anchors a coordinate system, the two coordinate systems overlap and pass through each other. Any small, well-defined event occurs at a three-number position (x_1, y_1, z_1) in one coordinate system and at a different three-number position (x_2, y_2, z_2) in the other coordinate system. The clock variable that Einstein added to these three-dimensional coordinate systems enters the discussion in Section II.

An object-anchored coordinate system is often called a reference frame. I am not going to use this term because it combines the idea of a coordinate system with a second idea in a way that can be confusing. The second idea is that of an *observation platform*, an object on which a person can sit or stand while observing the world. This is a useful idea, but it falls outside the scope of this paper. Avoiding the term “reference frame” with its ties to visual perception and visual perspective will help to keep the focus on coordinate systems. For the same reason, the word “observer” does not occur in this paper except in passages written by Einstein that I quote.

In his essay “What Is the Theory of Relativity?” Einstein made the following statement about object-anchored coordinate systems:

What has nature to do with our coordinate systems and their state of motion? If it is necessary for the purpose of describing nature, to make use of a coordinate system arbitrarily introduced by us, then the choice of its state of motion ought to be subject to no restriction; the laws ought to be entirely independent of this choice (general principle of relativity).¹

The context of this passage makes it clear that Einstein believed that object-anchored coordinate systems are “necessary for the purpose of describing nature.” He used the if/then construction not to register doubt about the “if” clause, but only to indicate that the “if” clause supports the “then” clause, which he also believed.

Was Einstein right about this? Is it necessary for the purpose of describing nature to make use of a coordinate system arbitrarily introduced by us? It is not obvious to me that this is so, and I have never seen a supporting argument or explanation. Clearly, one can successfully describe nature *to a large extent* without using coordinate systems. For example, without using coordinate systems one can do the following:

1. Enumerate the things that exist—tomatoes, mountains, protons—and point out ways in which they resemble and differ from each other.

2. Describe the structure or composition of things, such as the arrangement of organs in an animal or atoms in a molecule.
3. Describe sequences of events and causal connections between events.

Therefore, coordinate systems arbitrarily introduced by us are at most necessary for the purpose of describing *certain aspects* of nature, and an explanation of their indispensability should make clear what those aspects are. It should be possible to put the explanation in the following form: here are some aspects of nature that can only be described with the help of object-anchored coordinate systems, and here is why object-anchored coordinate systems are needed in order to describe them.

Since Einstein’s passage mentions “laws,” his view might be that it is specifically for the purpose of *stating laws of nature* that coordinate systems are necessary. If so, some clarification is called for, because there is no generally accepted definition of what a law of nature is. Everyone would agree, I think, that a statement of a law of nature must be a true statement about nature, expressed either in words or with a combination of words and mathematical notation. However, there is no consensus about how to draw a line between laws of nature and facts about nature that are not laws. Moreover, there is no practical need to draw such a line, because scientific research could go forward in the same familiar way if scientists stopped using the phrase “law of nature” and described their work as a search for significant facts about nature. The phrase “law of nature” is at least in part an honorary title that gets conferred on certain general facts that make a strong impression on those who work with them. Perhaps that is all it is.

Let us cast a wide net and consider the set of significant facts about nature that someone or other has called a law. Many of these have nothing to do with units of length and hence have no conceptual connection with object-anchored Cartesian coordinate systems. Examples are the laws of thermodynamics, the gas laws, and Mendel’s laws of inheritance. It is not possible, let alone necessary, to use object-anchored coordinate systems in statements of these laws. This brings us to the set of laws that do involve units of length. The following paragraph from the same essay suggests that this may be what Einstein had in mind when he wrote that coordinate systems arbitrarily introduced by us are necessary for the purpose of describing nature:

It has, of course, been known since the days of the ancient Greeks that in order to describe the movement of a body, a second body is needed to which the movement of the first is referred. The movement of a vehicle is considered in reference to the earth’s surface, that of a planet to the totality of the visible fixed stars. In physics the body to which events are referred is called the coordinate system. The laws of the mechanics of Galileo and Newton, for instance, can only be formulated with the aid of a coordinate system.²

If so, there's a problem. The all-important last sentence of this passage does not seem to be true. Newton's statements of his three laws of motion in his *Principia* make no use of coordinate systems, as far as I can tell, and the same is true of Galileo's writing on mechanics. Coordinate systems are useful for performing certain computations that are *based on* laws that were formulated by Galileo and Newton, but they do not seem to be necessary for stating the laws themselves.

But wait. There is an ambiguity, or at least a lack of clarity, in Einstein's use of the term "coordinate system," which must be taken into account. The next to last sentence of this passage equates a coordinate system with a "body." The laws of the mechanics of Galileo and Newton certainly do relate one body to another, so if that is all one means by a coordinate system, these laws can be said to require a coordinate system. If we read the passage this way, however, it says nothing about coordinate systems that are "arbitrarily introduced by us." Bodies are not arbitrarily introduced by us; they exist without our help. The arbitrariness and the human aspect of an object-anchored coordinate system consist in imaginatively attaching a set of three mutually perpendicular axes to a particular body that has been arbitrarily selected for that purpose. So, we have a choice between two equally unhelpful interpretations. If the last sentence of this passage is about object-anchored Cartesian coordinate systems, it is false. If it is about bodies as we find them in nature, it is true but irrelevant to the claim that it is necessary for the purpose of describing nature to use coordinate systems arbitrarily introduced by us. Either way, this passage gives no support to the claim that *coordinate systems arbitrarily introduced by us* are necessary for the purpose of describing nature.

Here is another possibility. Although the fundamental laws of the mechanics of Galileo and Newton can be stated without using coordinate systems arbitrarily introduced by us, these laws give rise to certain coordinate-system-dependent equations that one can also call laws. Coordinate systems arbitrarily introduced by us would then be necessary for the purpose of stating these secondary, coordinate-system-dependent laws. This may well be true, but it is important to note that it makes coordinate systems arbitrarily introduced by us "necessary for the purpose of describing nature" in only a rather weak sense. It makes them necessary, not in order to state fundamental laws (or facts) of nature, but only in order to state certain secondary laws (or facts) that are coordinate-system-dependent.

I conclude, tentatively, that the claimed necessity of using coordinate systems arbitrarily introduced by us for the purpose of describing nature is either illusory or considerably more limited than Einstein suggests. Coordinate systems arbitrarily introduced by us can be extremely useful for certain practical purposes, but they are not necessary in any strong sense for the purpose of describing nature. In general, if you make a creative effort to describe the aspect of nature that interests you in a coordinate-system-independent way, I believe you can do so. If you absolutely cannot get coordinate systems out of your description, you are probably describing a coordinate-system-dependent implication of

more fundamental laws (or facts) that you can state without using a coordinate system.

If this is right, there is reason to question the motive that drove Einstein's lifelong search for equations that describe the same phenomena no matter what object-anchored coordinate system their length variables are defined by. Here again is his stated justification for this search:

What has nature to do with our coordinate systems and their state of motion? If it is necessary for the purpose of describing nature, to make use of a coordinate system arbitrarily introduced by us, then the choice of its state of motion ought to be subject to no restriction; the laws ought to be entirely independent of this choice (general principle of relativity).¹

Einstein's thought here, as I understand it, is that any would-be law that took a different mathematical form in different object-anchored coordinate systems would be too dependent on something arbitrarily introduced by us to be a law of nature. My view is similar to Einstein's and yet anti-theoretical to it: any would-be law that requires object-anchored coordinate systems for its expression is by that fact alone too dependent on something arbitrarily introduced by us to be a *fundamental* law of nature. If it is in some sense a law, it is an implication of more fundamental laws that do not have this dependence. If my view is correct, I see no reason why the coordinate-system-dependent equations that a fundamental law gives rise to must be the same in all coordinate systems. A fundamental law that can be stated without using coordinate systems could have different mathematical implications in different object-anchored coordinate systems, reflecting their differing motions. Why not? Einstein and his followers are certainly welcome to search for equations that retain their form when transformed from one coordinate system to another, but if the most fundamental laws are coordinate-system-independent this requirement would seem to be a fetish without physical significance. Searching for equations with this kind of transformational symmetry would be like scouring documents for sentences with internal rhymes. You will probably find some important sentences that way, but you will overlook other sentences that are just as important.

II. THE EINSTEIN CLOCK VARIABLE DEFINED

The special theory of relativity uses object-anchored coordinate systems of a particular type, which Einstein characterized as coordinate systems "in which the Newtonian mechanical equations are valid."³ These are commonly called inertial coordinate systems. From here on, I discuss only inertial object-anchored coordinate systems.

One manifestation of the importance that Einstein attached to these coordinate systems was his addition of a fourth variable to them, which he defined by means of a spatially distributed array of clocks. Before Einstein, clocks were generally regarded as instruments to be used in a coordinate-system-independent way, like thermometers, microscopes, and most other laboratory devices. It took

someone with a very special regard for coordinate systems to think of *incorporating* clock readings into them. In the scheme Einstein devised, each inertial coordinate system has three mutually perpendicular axes marked off in units of length, as described in Section I, *plus* its own instance of a fourth variable that is defined by means of an array of clocks that are stationary with respect to the coordinate system's anchoring object. Many writers imagine these clocks attached to the imaginary nodes of an imaginary lattice made of imaginary rigid rods.

Einstein called this fourth variable "time." I will not. I am going to call it *the Einstein clock variable*. I set the word "time" aside because it conveys a sense of legitimacy that is incompatible with the criticism of this variable that I am going to make. Although perhaps not intended as such, calling this variable "time" is a branding maneuver that buttresses its reputation without argument. You will be in a better position to understand my analysis if you set this word aside and instead think "the Einstein clock variable."

The words "clock" and "time" are very closely linked in our mental ecosystem. In a word association game, each of them is likely to elicit the other. One might think, therefore, that the mere fact that the Einstein clock variable is a clock variable is sufficient reason to call it "time." This line of thought overlooks the important fact that the Einstein clock variable is based on *many* clocks, indefinitely many in fact. Indeed, each inertial coordinate system requires its own set of indefinitely many clocks to define its own instance of the Einstein clock variable, and there are indefinitely many inertial coordinate systems that have this requirement. So, a lot of clocks are involved and, as I will explain shortly, there are a lot of different ways in which one can use a lot of clocks to define a variable. It would be confusing nonsense to call every possible clock variable "time," and it would be arbitrary to call a particular clock variable "time" without first getting a sense of what the possibilities are. The easy thought "Clocks, therefore time" does not withstand scrutiny.

In later writings, Einstein sometimes used the phrase "the concept of time" in describing what he had done. Here are two examples:

An analysis of the concept of time was my solution.⁴

...the concept of time should be made relative, each inertial system being given its own special time.⁵

These descriptions are erroneous in two respects. They show that Einstein regarded his own work in a way that missed its essence. First of all, there is no such thing as *the* concept of time. There is *the* string of letters t-i-m-e (Z-e-i-t in German), which is associated with a vast complex of popular notions, scientific and philosophical theories, idiomatic expressions that people use without thinking, and so on. Second, the substance of what Einstein did was to set forth and advocate a new way of using clocks. In effect, he gave a novel answer to the question "How should we use clocks for the purpose of describing nature?" His answer to this question can be understood and assessed without bringing in the

word "time." The word "time" is associated with his proposal only because he chose to label the clock variable that he defined "time." In defining this variable, Einstein unquestionably invented a new concept. Since he labeled this concept "time," one might say that he invented *a new concept of time*. But he certainly did not invent, relativize, or analyze *the concept of time*.

Einstein wrote equations in which his clock variable is represented by the letter t . This practice is also prejudicial, because we have all been taught that " t stands for time." The word "time" and the variable t both appear in the passages written by Einstein that I will be quoting, but in my text the word "time" is absent and numbers produced by clocks are represented by the Greek letter Φ . As you read the pages that follow, I urge you to resist the subliminal hypnosis of the word "time" and the letter t , and focus on the actual characteristics of the Einstein clock variable. Let me repeat this in the imperative mood. *Focus on the actual characteristics of the Einstein clock variable.*

Einstein begins the project of defining his clock variable by mentioning certain things that one can do with an inertial coordinate system. One is to describe the position of a material point:

If a material point is at rest relative to this coordinate system, its position relative to the latter can be determined by means of rigid measuring rods using the methods of Euclidean geometry and can be expressed in Cartesian coordinates.⁶

Another is to describe the motion of a material point:

If we want to describe the *motion* of a material point, we give the values of its coordinates as a function of time. However, we should keep in mind that for such a mathematical equation to have physical meaning, we first have to clarify what is to be understood here by "time."⁷

Here Einstein could have said that we describe the motion of a material point by relating the values of its spatial coordinates to the numeric output of one or more clocks, and that questions therefore arise about the best way to use clocks for the purpose of describing motion. That would have focused the reader's attention on the substance of the inquiry—different ways of using clocks—without bringing in the word "time." Einstein is often credited with approaching his subject in an "operational" or "procedural" way. Here, the relevant operations and procedures concern the use of clocks. We do not need the word "time" to describe any of them.

Einstein's next step is to describe and reject the proposal that one use a single clock that is located at the coordinate system's origin and assign to any given event the numeric reading on this clock at the moment that light-borne news of the event reaches the origin. He rejects this proposal for the obvious reason that the clock is ticking while the news-bearing light is on the way to it from a distant event. The clock's reading when the light arrives is therefore different from (greater than) its reading when the light left the event.

He then proceeds to describe “a far more practical arrangement” that he advocates. This arrangement involves a spatially distributed array of clocks. Each clock is located at a fixed coordinate position and the clocks are set in a mutually coordinated way. I discuss the method of coordinating the clocks shortly. With enough mutually coordinated clocks distributed over a large enough region, every event of interest will occur near one of the clocks. One can then say that the clock-variable value for that event is the reading on the nearby clock when the event occurs. Einstein does not specify how closely the clocks should be spaced, presumably because this is a decision that depends on how much accuracy one requires. Each coordinate system must be equipped in this way with its own array of mutually stationary and mutually coordinated clocks in order to define its own instance of the Einstein clock variable.

A key deficiency of Einstein’s paper is that it mentions only these two possible ways of using clocks “to describe the motion of a material point”—the novel way that he advocates and the egregious way that he uses as a rhetorical foil. The fact that he mentions this alternative and says that the arrangement he advocates is “far more practical” shows that he recognizes the need to compare different possibilities and make a rational choice among them, but arguing that his array of mutually stationary and mutually coordinated clocks is better than an obviously horrible arrangement does not constitute a sufficient survey of the possibilities. One can imagine other possibilities, and perhaps one of them is superior to both the approach that Einstein rejects and the approach that he advocates.

If the material point belongs to a solid object, an obvious unmentioned alternative is to attach a clock to the object and use the readings on that clock when events of interest occur. A clock that is attached to the object will be right there next to every event in which the object participates. For example, if the object is a delivery truck and the events are the instances of the driver applying the parking brake when he stops to make a delivery, one could use the readings of a clock on the truck as the driver pulls the brake handle. A key characteristic of this method, which distinguishes it from both of the methods that Einstein mentions, is that it does not assign clocks to coordinate systems. With this method, an event will have different spatial coordinates in different coordinate systems but only one clock variable value, and the transformation from one set of coordinates to another will be accomplished via the purely spatial Galilean transformation that Einstein seeks to replace with the space-and-clock-combining Lorentz transformation. Yet Einstein makes no attempt to argue that for the purpose of describing nature four-variable, clock-incorporating coordinate systems are superior to three-variable coordinate systems used in conjunction with separately managed clocks. This straightforward and historically standard way of using clocks, which would seem to be a leading contender, is simply ignored. This is a remarkable omission, and equally remarkable is the consistent failure of those who write about Einstein’s paper to call it out.

Furthermore, if one adopts Einstein’s idea of using an array of clocks that are stationary relative to the coordinate

system’s anchoring object, one can generate alternatives to the Einstein clock variable by imposing other coordination conditions on the array. I mention some other possible coordination conditions in Section IV. Einstein did not discuss any coordination condition other than the one he advocated. In sum, he did not undertake the kind of comprehensive comparison of possibilities that would be necessary to demonstrate the superiority of his clock variable for the purpose of describing nature.

Here is Einstein’s complete description of his procedure for coordinating clocks:

If there is a clock at point A of space, then an observer located at A can evaluate the time of the events in the immediate vicinity of A by finding the clock-hand positions that are simultaneous with these events. If there is also a clock at point B—we should add, “a clock of exactly the same constitution as that at A”—then the time of the events in the immediate vicinity of B can likewise be evaluated by an observer located at B. But it is not possible to compare the time of an event at A with one at B without a further stipulation; thus far we have only defined an “A-time” and a “B-time” but not a “time” common to A and B. The latter can now be determined by establishing by definition that the “time” needed for light to travel from A to B is equal to the “time” it needs to travel from B to A. For, suppose a ray of light leaves from A toward B at “A-time” t_A , is reflected from B toward A at “B-time” t_B , and arrives back at A at “A-time” t'_A . The two clocks are synchronous by definition if

$$t_B - t_A = t'_A - t_B.$$

We assume that it is possible for this definition of synchronism to be free of contradictions, and to be so for arbitrarily many points, and that the following relations are therefore generally valid:

1. If the clock in B is synchronous with the clock in A, then the clock in A is synchronous with the clock in B.
2. If the clock in A is synchronous with the clock in B as well as with the clock in C, then the clocks in B and C are also synchronous relative to each other.

With the help of some physical (thought) experiments, we have thus laid down what is to be understood by synchronous clocks at rest that are situated at different places, and have obviously obtained thereby a definition of “synchronous” and of “time.” The “time” of an event is the reading obtained simultaneously with the event from a clock at rest that is located at the place of the event and that for all time determinations is in synchrony with a specified clock at rest.⁸

Here is a description of this procedure that does not use the word “time” or the variable symbol t . For any given inertial coordinate system, an instance of the Einstein clock variable is defined by a spatially distributed array of clocks that are at rest relative to the object that anchors the coordinate system. The clocks are mutually coordinated using light signals in the following way. Clocks A and B are set so that if a light pulse goes straight from clock A to clock B and is reflected straight back to clock A, and clock A reads Φ when the light pulse leaves it and $\Phi + \Delta$ when the light pulse returns to it, then clock B reads $\Phi + 0.5\Delta$ when the light pulse is reflected from it. Put another way, the clocks are set so that the three clock readings encountered by the light pulse on its round trip define two numerically equal intervals of 0.5Δ .

Another word that appears in Einstein’s description but is absent from my description is “synchronous.” I have said several times that Einstein specified a procedure for *coordinating* clocks, but I have not said that he specified a procedure for *synchronizing* clocks. “Coordinating” is a safe and unproblematic word in this context. To coordinate clocks is merely to connect the clock settings in some way or other, so that they are not independent of each other. Einstein’s procedure clearly does that. “Synchronizing” is more specific than “coordinating,” but in a way that is not obvious. Hence its use in connection with Einstein’s clock-coordination procedure raises questions. We need to examine Einstein’s use of this word.

In the passage, Einstein *defines* “synchronous” to mean satisfying the clock-coordination condition that the passage describes. “The two clocks are synchronous by definition,” he writes, if they satisfy the specified condition. But note that the word “synchronous” is an element of our mother tongue, and as such it already has a meaning apart from this definition that Einstein gives it. A natural question, then, is how Einstein’s definition of “synchronous” is related to the mother-tongue meaning of this word. Einstein never discussed this question. I know of nothing he wrote that suggests he even thought about this question, although of course he might have. In any case, it is easy to demonstrate that Einstein’s definition of “synchronous” is at least somewhat different from the mother-tongue meaning of “synchronous.” Consider two airplanes that cross paths in flight. For example, suppose one plane is flying east from Paris to Vienna, another is flying south from Berlin to Rome, and they cross paths over the Alps. Passengers in either plane can look out the window and see the other plane, a kilometer or so above or below them, headed in a different direction. Is it conceivable that a clock on one of these planes is running synchronously with a clock on the other plane? If “synchronously” is used here with its mother-tongue meaning, then of course this is conceivable. Whether nature permits it is another question, but we can certainly imagine clocks on these two planes running “synchronously.” However, if “synchronously” is used with Einstein’s definition, this is not conceivable, because his definition involves the round trip of a light pulse between two clocks *that are at a constant distance from each other*. This condition cannot be met by two clocks that are crossing paths, so it is logically impossible for these clocks to

run synchronously in the sense of Einstein’s definition. In at least this respect, Einstein’s definition of “synchronous” differs from the mother-tongue meaning.

This practice of associating a novel technical definition with a word that has a mother-tongue meaning can set the stage for confusion and confusion-generated mistakes. To see how this can happen, consider the more concrete example of the word “obese.” As a boy, I learned to use this word in a rough-and-ready way based on people’s visible bodily proportions. The medical profession in the United States uses a statistic called the body mass index (BMI), which is a function of height and weight, as a tool for assessing an individual’s health risks. Some decades ago, a committee of medical professionals decided to label numerical ranges on the BMI scale with everyday words. As part of this labeling the committee decreed that anyone with a BMI of 30 or more was “obese.” In the wake of this stipulation, the word “obese” has two senses. Someone not acquainted with the BMI definition of “obese” who looks at a person whose BMI is in the low 30’s would never call that person obese based on the rough-and-ready mother-tongue meaning of the word. Some words that come to mind to describe a person with a BMI of 30 or 31 are “plump,” “chubby,” and “stout.” Obesity in the mother-tongue sense starts higher on the BMI scale, perhaps in the mid 30’s. The co-existence of these two senses of “obese” has two significant consequences. First, a person can be obese in one sense but not in the other sense; one must therefore maintain an awareness of the two senses and be careful not to assume that both senses apply whenever one does. Second, it is very difficult to hear the word “obese” without thinking of the mother-tongue meaning that you learned as a child, even if someone intends the word only in its BMI sense. The stage is thus set for confusion and misunderstanding.

The case of “synchronous” does not match the case of “obese” in every detail, but the same general points apply. First, two clocks can be synchronous in one sense but not in the other sense. The airplane example shows that two clocks can be mother-tongue synchronous without being synchronous by Einstein’s definition, and later in the paper it will emerge that the opposite discrepancy is also possible: Two clocks that are synchronous by Einstein’s definition need not be mother-tongue synchronous. Second, it is very difficult to read the word “synchronous” without having a mother-tongue reaction to it, even if that reaction is inappropriate. One must be constantly on guard against making fallacious inferences based on treating the two senses of “synchronous” as one.

I have noted already that Einstein does not discuss the relation between his definition of “synchronous” and the mother-tongue meaning of “synchronous.” In particular, he does not demonstrate any connection between his definition and the word’s mother-tongue meaning. He does not *show* that clocks that have been coordinated with one another in the way that he specifies are synchronous in some independent sense. To say that two clocks that are mutually coordinated by Einstein’s method are synchronous by Einstein’s definition is merely to say that they are mutually coordinated by Einstein’s method. It is a tautology, not a thesis. The

word “synchronous” thus functions as another element in the branding campaign that includes the word “time” and the letter t . It adds nothing of substance to Einstein’s paper. In particular, it gives no reason why Einstein’s proposed method of managing clocks is superior to other methods of managing clocks for the purpose of describing nature. “Synchronous” is another word that should be set aside in order to get the clearest possible view of the Einstein clock variable’s actual characteristics.

All of this applies as well to the word “simultaneous.” The section of Einstein’s paper that contains the passages I have quoted is titled “Definition of Simultaneity.” This is a bit odd, because the section gives definitions of “time” and “synchronous,” but no definition of “simultaneity.” It does contain several occurrences of the word “simultaneous,” but these are seemingly meant to be understood in the word’s mother-tongue sense. The most plausible way to fill this gap, which is also the way both followers and critics of Einstein generally do fill it, is to suppose that Einstein’s explicit definition of “synchronous” clocks entails the following definition of simultaneity for two events. Two events A and B are simultaneous with respect to a given 4-variable inertial coordinate system if the following three conditions are satisfied:

1. Each event happens next to a clock that is part of the array that defines the coordinate system’s clock variable.
2. The clocks are “synchronous” in the sense defined by Einstein.
3. The reading on the clock next to event A when event A occurs is numerically equal to the reading on the clock next to event B when event B occurs.

This definition of “simultaneous” rides piggyback on Einstein’s definition of “synchronous,” and has no particular relation to the mother-tongue meaning of the word “simultaneous.” It is another element in the branding campaign that should be set aside in order to get the clearest possible view of the Einstein clock variable’s actual characteristics.

Time and t , synchronous and simultaneous—we need to strip away all of this hypnotic verbiage and focus on the Einstein clock variable’s actual characteristics. Is it the best way to use clocks for the purpose of describing nature?

III. THE EINSTEIN CLOCK VARIABLE DISSECTED

Here is a simple principle regarding the behavior of light:

In a vacuum, if two light pulses are traveling neck-and-neck in precisely the same direction, they will continue to travel neck-and-neck.

Let us call this *the neck-and-neck principle*. I have never seen a statement of this principle, perhaps because it is too simple to attract attention, or perhaps because it makes no use of object-anchored coordinate systems, which so many physicists seem to be tied to. In any case, all relevant evidence supports it, and I will assume that it is true.

Although the neck-and-neck principle is easy to state and easy to understand, it is not trivial. If two horses are running neck-and-neck in precisely the same direction, it is very likely that they will not remain neck-and-neck for long. This is because horses are extremely complex systems that differ from one another in many ways, some of which affect their running ability. Light pulses are not as complex as horses, but they do have significant internal structure and they can differ from one another in various ways, such as frequency, wavelength, and polarization. One can imagine that one or more of these structural differences affects their speed of travel. The neck-and-neck principle says this is not the case. Whatever structural differences there may be between two neck-and-neck light pulses, the pulses will continue to travel neck-and-neck.

Two neck-and-neck light pulses might have come from different sources that were in motion relative to each other. Since the neck-and-neck principle does not mention source motion, it also says, implicitly, that differences of source motion are irrelevant. In a vacuum, two neck-and-neck light pulses continue to travel neck and neck, period.

Einstein’s “second postulate” concerning the speed of light says, in part, that the speed of light does not depend on the motion of its source. Einstein cites the behavior of double stars as evidence for this claim.⁹ A double star is a system of two stars that together orbit their center of mass, like a giant merry-go-round in the sky. For a double star that has the earth in or near its orbital plane, there are regular occasions when the two stars are equidistant from the earth, with one of them approaching the earth and the other moving away from the earth. From the clarity with which astronomers are able to view these systems one can infer that light pulses that leave the two stars headed for the earth when the stars are equidistant from the earth arrive on earth at the same time. Since the light pulses travel the same distance, it follows that they travel at the same speed. This behavior is also evidence for the neck-and-neck principle: light pulses that leave the stars neck and neck and arrive on earth neck and neck are presumably neck and neck throughout the trip. It is interesting to compare the neck-and-neck principle with Einstein’s second postulate in detail, but doing so is not essential to my argument, so let’s move on.

I will now apply the neck-and-neck principle to a simple thought experiment. Figure 1 shows two identical rocket ships, A and B, which are passing each other with a uniform relative velocity, headed in opposite directions. Ship A is headed to the right, ship B to the left, as indicated by their nose cones.

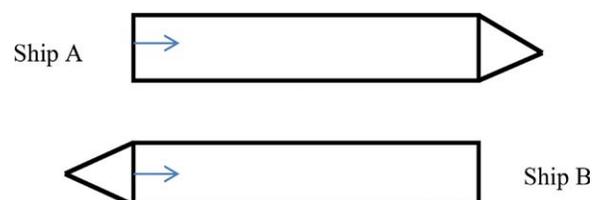


FIG. 1. (Color online) Two light pulses emitted on the left inside passing rocket ships.

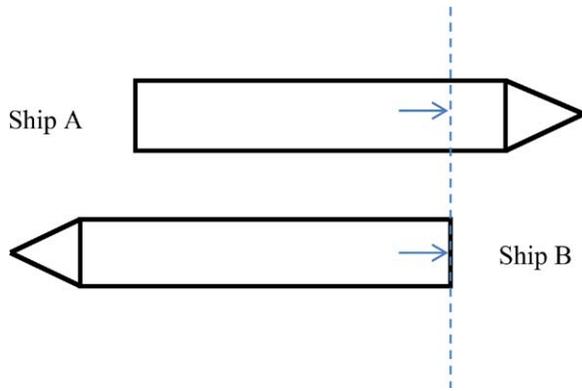


FIG. 2. (Color online) Rightward-propagating light pulses reach the tail clock of ship B.

Inside each ship there are two clocks, one at the nose and one at the tail. All four clocks are identical. When the two ships are precisely abreast of each other, as shown in Fig. 1, light pulses (represented by the two rightward-pointing arrows) are emitted from the two clocks on the left—the tail clock of ship A and the nose clock of ship B. According to the neck-and-neck principle, these two light pulses will travel neck and neck to the right. The situation is very similar to light pulses that simultaneously leave the two, oppositely moving members of a double star, headed for the earth. Meanwhile, the two clocks on the right will be separating from each other due to the relative velocity of the ships. It follows that both light pulses will reach the ship B clock on the right first, as shown in Fig. 2.

Light has a wave structure. Suppose that these light pulses are monochromatic pulses produced by identical devices. Each pulse will advance by continuously generating electric and magnetic fields at its leading edge, in accordance with the wave equation for electromagnetic radiation. Let us call one complete repeating unit of this sinusoidal wave a wave cycle. When they reach the tail clock of ship B, as shown in Fig. 2, each light pulse will have generated at its leading edge, in rapid succession, a certain number of wave cycles, b . The light pulse in ship A will generate an additional number of wave cycles before it reaches the nose clock of ship A, for a total of $a > b$ wave cycles.

The same reasoning applies if we consider two light pulses starting from the clocks on the right, as shown in Fig. 3.

According to the neck-and-neck principle, these two light pulses will travel neck and neck to the left. Meanwhile, the two clocks on the left will be separating from each other due to the relative velocity of the ships. It follows that both

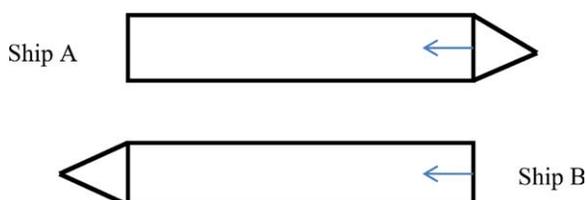


FIG. 3. (Color online) Two light pulses emitted on the right inside passing rocket ships.

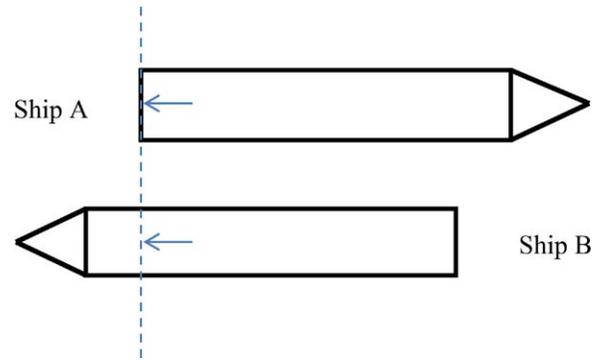


FIG. 4. (Color online) Leftward-propagating light pulses reach the tail clock of ship A.

light pulses will reach the ship A clock on the left first, as shown in Fig. 4.

Suppose that these light pulses are monochromatic pulses produced by devices identical to those that produce the rightward-propagating pulses. When they reach the tail clock of ship A, as shown in Fig. 4, each will have generated at its leading edge, in rapid succession, a certain number of wave cycles, α . The light pulse in ship B will generate an additional number of wave cycles before it reaches the nose clock of ship B, for a total of $\beta > \alpha$ wave cycles.

Notice that I am using Latin letters to describe the rightward-propagating pulses and Greek letters to describe the leftward-propagating pulses. Notice too that I am using the first letter of the alphabet for the ship A light pulses and the second letter of the alphabet for the ship B light pulses. These conventions are intended to make the reasoning below easy to follow.

If a light pulse makes a round trip in ship A, starting and ending at one clock and reflecting off the other clock, it will generate $a + \alpha$ wave cycles in the course of the round trip. Likewise, if a light pulse makes a round trip in ship B, starting and ending at one clock and reflecting off the other clock, it will generate $b + \beta$ wave cycles in the course of the round trip. It is easy to show that the conjunction ($a = \alpha$ and $b = \beta$) is logically impossible:

From the discussion so far, we know that $a > b$ and $\beta > \alpha$.

Suppose that $a = \alpha$. Then we can substitute either of these for the other in either of the preceding inequalities.

If we substitute a for α in the second inequality, we get $a > b$ and $\beta > a$.

Chaining these two inequalities yields $\beta > a > b$.

Therefore, $b \neq \beta$.

In like manner, one can suppose that $b = \beta$ and deduce that $a \neq \alpha$.

The conclusion is that at least one of the light pulses generates more wave cycles on one leg of its round trip than on the other. Quite possibly this is the case for both light pulses, but it must be the case for at least one of them.

A monochromatic light pulse is a kind of clock; each wave cycle that it generates constitutes one tick. Moreover, it is an extremely accurate clock; as far as we know there is no difference at all between the successive wave cycles of a monochromatic electromagnetic wave in a vacuum. I will

call a clock of this kind an *electromagnetic wave clock*. Unlike the manufactured objects that we are used to calling clocks, an electromagnetic wave clock does not have a counter or a numeric display. Therefore, it cannot *show us* how many wave cycles it has generated since it started ticking. Nevertheless, if light has the wave structure it is said to have, each pulse will generate a definite number of wave cycles on each leg of its round trip. Imagine a massive oak tree in midsummer. One does not have to count its leaves in order to know that it has a definite number of leaves. Another example, with more similarities to a propagating light pulse, is a flying goose. A goose takes off from a pond and rhythmically flaps its wings until it lands on another pond nearby. One does not have to count the flaps in order to know that a definite number of them occurred while the goose was in the air. Likewise, one does not have to count the wave cycles generated by a monochromatic light pulse between two events in order to know that it has generated a definite number of wave cycles.

Now we come to the crux of the matter. At least one of the light pulses generates more wave cycles on one leg of its round trip than on the other. Thus, at least one of the electromagnetic wave clocks ticks more times on one leg of its round trip than on the other. This fact puts the electromagnetic wave clock at odds with the Einstein clock variable. Einstein’s definition specifies that the value of the clock variable defined jointly by two clocks that are at a fixed distance from each other increases by exactly the same amount during each leg of a light pulse’s round trip, *regardless of the motion of the clocks*. But in general, an electromagnetic wave clock does not generate the same number of wave cycles on each leg of such a round trip. In general, it generates more wave cycles on one leg than on the other, with the magnitude of this difference *depending on the motion of the clocks*.

It is possible for the clocks at the two ends of a rocket ship, or any pair of clocks separated by a fixed distance, to agree with an electromagnetic wave clock that makes a round trip between them. Suppose that a monochromatic light pulse makes a round trip from clock Q to clock R and back to clock Q. Suppose that the entire round trip comprises k wave cycles, and that pk wave cycles occur on the first leg of the trip and the remaining $(1-p)k$ wave cycles occur on the return leg, where p is a fraction between 0 and 1. If clock Q reads Φ at the start of the round trip and $\Phi + \Delta$ at the end of the round trip, the two clocks will agree with the electromagnetic wave clock if clock R reads $\Phi + p\Delta$ when the light pulse is reflected from it. It is true that we have no procedure for setting clock R to ensure that this condition is met, because we have no way to count the number of wave cycles that have been generated by the electromagnetic wave clock. But it is possible for the pair of clocks Q and R to satisfy this condition as a matter of fact.

If clock R reads $\Phi + 0.5\Delta$ when the light pulse is reflected from it, as Einstein prescribes, then its reading is either ahead of (if $p < 0.5$) or behind (if $p > 0.5$) the reading that would put the clocks in agreement with the electromagnetic wave clock. Since the electromagnetic wave clock is the most accurate clock we know of, there is no way to

justify having clocks Q and R satisfy Einstein’s $\Phi + 0.5\Delta$ condition instead of the electromagnetic wave clock’s $\Phi + p\Delta$ condition. Einstein’s prescription can seem acceptable when presented in Einstein’s manner—mentioning only one pair of man-made clocks, ignoring the clock-like nature of light, and using the words “time” and “synchronized” to brand his proposal. But if you consider two pairs of man-made clocks in relative motion, if you take account of the clock-like nature of light, and if you are on your guard against marketing stunts, you can see that Einstein’s prescription violates a fundamental natural constraint on the management of clocks. Of course, in many circumstances Einstein’s $\Phi + 0.5\Delta$ condition will be a very good approximation to the electromagnetic wave clock’s $\Phi + p\Delta$ condition, but it is only an approximation.

Suppose now that instead of the two rocket ships depicted in the four figures, we have an indefinitely large number of identical rocket ships, each of which is in uniform motion relative to all the others. Assume that in one of them, let us say ship A, the light pulse generates the same number of wave cycles on each leg of its round trip: $a = \alpha$. Then, by the reasoning just presented, the light pulse in every other ship generates different numbers of cycles on the two legs of its round trip: $b \neq \beta$ in ship B, $d \neq \delta$ in ship D, $e \neq \varepsilon$ in ship E, and so on. This shows that in general $p \neq 0.5$; a rocket ship for which $p = 0.5$ is in a very special condition.

The instance of the Einstein clock variable that belongs to a given 4-variable inertial coordinate system is defined by a spatially distributed array of clocks that extends indefinitely far in all directions from the origin. Let the clock at the origin be C_0 and let $C_0, C_1, C_2, C_3, \dots$ be a row of equally spaced clocks stretching along the x-axis. We set C_0 to a chosen reading, and then compare two conditions for coordinating the other clocks with C_0 —Einstein’s $\Phi + 0.5\Delta$ condition and the electromagnetic wave clock’s $\Phi + p\Delta$ condition. Suppose that $p \neq 0.5$ as is typically the case. Then for every other clock in the row there will be a discrepancy between the setting that satisfies Einstein’s $\Phi + 0.5\Delta$ condition and the setting that satisfies the electromagnetic wave clock’s $\Phi + p\Delta$ condition. For clock C_i , the discrepancy will be

$$(\Phi + 0.5\Delta_i) - (\Phi + p\Delta_i) = (0.5 - p)\Delta_i, \tag{1}$$

where Δ_i is the change in the reading on clock C_0 during the round trip of a light pulse from clock C_0 to clock C_i and back to clock C_0 . Since the change in the reading on clock C_0 during the round trip is proportional to the distance of the round trip, the discrepancy for a given clock is proportional to its distance from clock C_0 .

For any given moment, the set of all the readings on all the clocks in the array defines a linear function of the three spatial coordinate variables. Coordinating the clocks to satisfy Einstein’s $\Phi + 0.5\Delta$ condition yields one linear function, which we may call $f(x, y, z)$. Coordinating the clocks to satisfy the electromagnetic wave clock’s $\Phi + p\Delta$ condition yields a different linear function, which we may call $g(x, y, z)$. If the electromagnetic wave clock is the high-quality clock that I claim it is, then $g(x, y, z)$ is the constant function: an electromagnetic wave clock traveling from one

man-made clock to another through the array would be in agreement with each man-made clock that it visits. It follows that $f(x, y, z)$ is not the constant function: an array of clocks that is coordinated to satisfy Einstein's $\Phi + 0.5\Delta$ condition will in general define a linear function that has a nonzero gradient. The gradient will be extremely close to zero because the speed of light is so great, and it will be especially close to zero if the motion of the clock array is such that p is very close to 0.5, but it will not be precisely zero except in the special case where $p = 0.5$. A useful although imperfect analogy is atmospheric pressure as a function of location in the atmosphere. At every location in the atmosphere, the pressure has a certain value, and these values decline steadily—although not precisely linearly—as one ascends from the earth's surface. Analogously, an instance of the Einstein clock variable has a value at every location in the object-anchored coordinate system that it belongs to, and these values slope ever so gently in a direction that depends on the motion of the anchoring object.

A clock variable that has this characteristic is not ideally suited for the purpose of describing nature. If you compute the duration of a trip from one clock to another in an Einstein clock array by subtracting the departure clock's reading at the moment of departure from the arrival clock's reading at the moment of arrival, the number you get will be artificial in the same sense as the number you get if you compute the duration of a flight from New York to London by simply subtracting the New York airport departure reading from the London airport arrival reading. The reason for the artificiality is that you are using clocks that are offset relative to each other as if they are not offset. You are ignoring the nonzero spatial gradient of the function defined by the clock readings. Of course, the offset between two clocks in an Einstein clock array will in general be extremely small, as compared with the multiple-hour offsets between airport clocks that are set for different time zones. But both computations have the same sort of artificiality.

If you find the preceding criticism of the Einstein clock variable unconvincing, consider the following closely related argument, which has a weaker but still significant conclusion.

It is obviously possible to construct a spatially distributed array of clocks that defines a clock variable $h(x, y, z)$ having a nonzero spatial gradient. For example, place a clock on the ground floor of a high-rise building, place an identical clock set ahead by a small increment on the second floor, place an identical clock set ahead by twice that small increment on the third floor, and so on to the roof and beyond. Since it is physically possible to construct an array of clocks having a nonzero spatial gradient, it is logically possible that Einstein's $\Phi + 0.5\Delta$ prescription for setting clocks produces an array that has that property. This was not Einstein's intent, of course, but one can imagine that it is the unintended result. *Can you rule out this possibility?* Can you construct a convincing argument that no instance of the Einstein clock variable has a nonzero spatial gradient? In other words, can you show that every instance of the Einstein clock variable has a spatial gradient of zero in all directions, regardless of the motion of the object that anchors the coordinate system?

Imagine Einstein giving a lecture in which he explains his procedure for coordinating clocks. At the end he asks "Are there any questions?" A skeptical listener challenges him as follows:

You say that clocks that have been coordinated by your method are synchronous *by definition*. I worry that this use of the word "synchronous" amounts to a cover-up of your failure to rule out the possibility that your coordination procedure produces a clock variable that has a non-zero spatial gradient. You did not discuss this possibility. Have you thought about it? Are you hiding it from yourself? Do you have an argument that rules it out?

What could Einstein say in reply? What can anyone say in reply? If you cannot produce a cogent argument, then you must grant that the Einstein clock variable *might* have a nonzero spatial gradient, even if my argument has not convinced you that it does. This alone is a serious problem. If it would be a perverse and artificial practice to use a clock variable that you know has a nonzero spatial gradient for the purpose of describing nature, then it would be irresponsible to use a clock variable that for all you know might have this property. The mere possibility that the Einstein clock variable has this artificial property is a reason not to use it.

IV. OTHER SPATIALLY SLOPING CLOCK VARIABLES

Here is another argument that reinforces the conclusion of Section III that the Einstein clock variable has a nonzero spatial gradient. Einstein furnishes each inertial coordinate system with an instance of his clock variable. Let us instead furnish each inertial coordinate system with an instance of a clock variable that is deliberately designed to have a nonzero spatial gradient. For example, the clocks belonging to one coordinate system might have readings that increase by one minute for each 100 km along the x axis, the clocks belonging to another coordinate system might have readings that increase by one second for each 1000 km along the x axis, and so on. It is easy to see that these coordinate systems are related to one another in ways that are eerily isomorphic to the principal claims of the special theory of relativity. If in addition we play the word game and say that "by definition" all the clocks that belong to a given coordinate system are synchronous with one another and collectively define time for that coordinate system, these results can be expressed in the same sensational manner that the special theory of relativity made famous. Following is a survey of how this works.

Multiple time values for a single event. Einstein sets the stage for this multiplicity by abandoning the traditional independence of clocks from coordinate systems and instead furnishing each coordinate system with its own instance of a clock variable. If these clock-variable instances have nonzero spatial gradients and the coordinate systems are in relative motion, then of course a given event is likely to have different clock-variable values in different coordinate systems. If you call all these clock-variable values time, then

the event has different time values in different coordinate systems.

Lorentz-like transformation between coordinate systems. Consider the standard set-up for the derivation of the Lorentz transformation: one coordinate system moves along the x axis of another coordinate system after a moment when the $(0, 0, 0, 0)$ points of the two coordinate systems coincide. Any subsequent event will have clock-variable values in the respective coordinate systems that differ by a term that is proportional to the distance that the two coordinate systems have traveled relative to each other. The reason is that this distance determines how far a clock that belongs to one coordinate system has climbed up (or down) the nonzero spatial gradient of the other coordinate system's clock variable. This dependence of the clock-variable transformation on a spatial variable is the distinctive characteristic of the Lorentz transformation. If you call the clock variable time, then you can describe this dependence in sensational terms as the merging of space and time.

One of the first people to express himself in this way was Hermann Minkowski, who waxed poetic in the following famous statement:

From now on space by itself and time by itself will recede completely to become mere shadows and only a type of union of the two will stand independently on its own.¹⁰

This thought is the result of projecting the characteristics of coordinate systems that have clock-variable instances with a nonzero spatial gradient onto coordinate-system-independent nature. The “type of union” that Minkowski ascribes to nature is manufactured in human imagination by defining a clock variable that is a linear function of the spatial locations of the clocks. Treating that mentally manufactured “spacetime” as a discovered feature of the universe is like taking at face value a person's weird image in a distorting mirror, or viewing a landscape through a flawed window whose presence you are unaware of.

Relativity of simultaneity and relativity of chronological order. Because the clock-variable instances of different coordinate systems have spatial gradients that differ in direction and steepness, it can easily be the case that two widely separated events are ordered differently by the clock-variable instances of different coordinate systems. Event A and event B might have the same clock-variable value in one coordinate system but different clock-variable values in another coordinate system. Likewise, event A might have the smaller clock-variable value in one coordinate system and the larger clock-variable value in another coordinate system. If you say that all the clocks that belong to one coordinate system are synchronous and together define time for that coordinate system, then you can describe these facts in sensational terms as the relativity of simultaneity and the relativity of before and after.

Time dilation. If a clock that belongs to one coordinate system moves through the array of clocks that defines the clock variable of another coordinate system in a direction of positive gradient, each clock that it passes will be slightly ahead of the last clock that it passed. If you say that the

clocks in the spatially sloping array are synchronous and together define time, then you will be led to say that “the moving clock is running slow” relative to the time defined by the array.

Length contraction. If a long straight rod moves through an array of clocks in a direction of positive gradient and you mark the location of each end of the rod when the clock that it is next to has a certain reading Φ , then you will mark the location of the trailing end of the rod later than you will mark the location of the leading end. As a result, the distance between the marks will be less than the rest length of the rod. If you say that the two marks were made simultaneously because they were made in association with the same numerical clock reading Φ on the respective clocks, then you will be led to say that the distance between the marks is the length of the rod as measured in the coordinate system to which the clocks belong. The fact that the distance between the marks is less than the rod's rest length can then be expressed with the sensational claim that the rod is contracted in this coordinate system.

One cannot derive Einstein's exact formulas in this way because the clock variables I have used here are characterized by nonzero spatial gradients of arbitrary magnitude that are unrelated to round-trip light signals. To derive Einstein's formulas you need not just any clock variable that has a nonzero spatial gradient, but a clock variable that has the precise nonzero spatial gradient that results from Einstein's $\Phi + 0.5\Delta$ clock-coordination condition. However, every significant qualitative feature of Einstein's formulas has a counterpart in these derivations. Given that the qualitative essence of all these features can be derived almost trivially using clock variables that have any old nonzero spatial gradients—and *precisely because they have nonzero spatial gradients*—it would be quite a coincidence if they could also be derived, for some wholly different reason, from a clock variable that does not have a nonzero spatial gradient. This constitutes a strong circumstantial case that the essential generator of Einstein's results, lurking just beneath his algebra, is the fact that he (inadvertently) furnished his coordinate systems with clock-variable instances that have nonzero spatial gradients.

An interesting corollary of this argument is that some of the sensational claims of the special theory of relativity are arguably true, if you co-opt the words “time,” “synchronous,” and “simultaneous” and hitch them to the Einstein clock variable in the way that Einstein does. This fact may help to explain why so many physicists have seen fit to defend these claims. The trouble is that any claims whose truth depends on tying these words to the Einstein clock variable are not truths about nature. They are truths about the relationships between inertial coordinate systems that are furnished with clock-variable instances that have nonzero spatial gradients.

V. THE VELOCITY OF LIGHT

Einstein defines his clock variable in Section 1 of his paper. In the first paragraph of Section 2, he writes the following:

Each ray of light moves in the coordinate system “at rest” with the definite velocity V independent of whether this ray of light is emitted by a body at rest or a body in motion. Here,

$$velocity = \frac{\text{light path}}{\text{time interval}}$$

where “time interval” should be understood in the sense of the definition in Section I.¹¹

In this definitional equation, the term “time interval” in the denominator refers to the result of subtracting the beginning-of-trip reading on a clock located where the light ray’s trip begins from the end-of-trip reading on a clock located where the light ray’s trip ends, the clocks having been previously set so that they satisfy Einstein’s $\Phi + 0.5\Delta$ condition. If these clocks are instead set so that they satisfy the $\Phi + p\Delta$ condition, which aligns them with an electromagnetic wave clock that travels from one man-made clock to the other and back, the denominator will be different and thus the result of the computation will be different. Einstein’s use of the word “velocity” for the result of this computation is thus as misleading as his use of the phrase “time interval” for the denominator; one redefinition of a familiar word based on the Einstein clock variable paves the way for another. I will instead call the result of the computation that Einstein specifies here *levocity*. This coinage is designed to indicate two things. First, the resemblance of the character string “levocity” to the character string “velocity” is meant to indicate that the quantity it designates is *velocity-like*, in that it is the result of dividing a distance by a number obtained from clocks. Second, the swapping of the first two consonants is meant to indicate that the designated quantity inherits the artificiality of the spatially sloping Einstein clock variable that is used to determine the “time interval.”

Einstein uses the term “the principle of the constant velocity of light” for the generalization of the quoted assertion to all of his 4-variable inertial coordinate systems: each ray of light moves in *every such coordinate system* so as to satisfy the specified condition. A better name for this generalization is “the principle of the constant *levocity* of light.” According to this principle, light has the same *levocity* in all 4-variable inertial coordinate systems in which clocks are set so that they satisfy Einstein’s $\Phi + 0.5\Delta$ condition. I do not know whether this principle is true or not, but I do know that it should not be of interest to anyone who considers a coordinate system that has a spatially sloping clock variable to be inappropriate for the purpose of describing nature.

What should be of interest to everyone, on the other hand, is whether light has a constant velocity in a sense that uses the electromagnetic wave clock or man-made clocks that are in harmony with the electromagnetic wave clock. There is good reason to believe that there is such a sense in which light has a constant velocity. The relevant sense is illustrated by the neck-and-neck principle that I introduced at the beginning of Section III: two light pulses that are traveling neck-and-neck in the same direction will continue to travel neck and neck. Each light pulse advances at the speed

of one wavelength per wave cycle period, and these speeds are identical for light of all wavelengths because it is a fact about electromagnetic radiation that wavelength and wave cycle duration vary together in strict proportion. If the wavelength of one light pulse is n times the wavelength of another light pulse, then the ratio of their cycle durations is also n . The statement that light has a constant velocity in this sense makes no use of the Einstein clock variable, and indeed it makes no use of object-anchored coordinate systems. It is simply a statement about the behavior of light in a natural vacuum, where there are no objects to obstruct the light and no coordinate systems to distract the mind.

One reason for the widespread acceptance of Einstein’s claim that light has a constant velocity in every inertial coordinate system is that it resembles this other claim that light has a constant velocity in a coordinate-system-independent sense. Chapter VII of Einstein’s book *Relativity: The Special and the General Theory* begins with the following paragraph, to which I have no objection:

There is hardly a simpler law in physics than that according to which light is propagated in empty space. Every child at school knows, or believes he knows, that this propagation takes place in straight lines with a velocity $c = 300,000$ km/sec. At all events we know with great exactness that this velocity is the same for all colours, because if this were not the case, the minimum of emission would not be observed simultaneously for different colours during the eclipse of a fixed star by its dark neighbor. By means of similar considerations based on observations of double stars, the Dutch astronomer De Sitter was also able to show that the velocity of propagation of light cannot depend on the velocity of motion of the body emitting the light. The assumption that this velocity is dependent on the direction “in space” is in itself improbable.¹²

The trouble starts when Einstein continues as follows:

Of course we must refer the process of the propagation of light (and indeed every other process) to a rigid reference-body (co-ordinate system). As such a system let us again choose our embankment...

No, no, no! This is not something that we *must* do. The comforting “Of course” helps to cover up the wrongness of the “must.” We can easily think about the propagation of light without introducing any object-anchored coordinate system. Indeed, the entire paragraph I have just quoted can be read as an expression of such thoughts, even if these are not the thoughts that Einstein had when he wrote it. The axes of object-anchored coordinate systems were prison bars for Einstein’s imagination, which kept him from having, or at least from pursuing, thoughts about space, light, and motion in which coordinate systems play no part.

Of course, we *may* refer the process of the propagation of light to an object-anchored coordinate system. It is not a mistake to do this. But then we face the question of how to

use clocks in conjunction with such a coordinate system. If we use clocks that are aligned with the electromagnetic wave clock, light propagates in different inertial coordinate systems with different velocities, in accordance with the different motions of the coordinate systems, as explained in Section III. If we use instances of the Einstein clock variable, which are at odds with the electromagnetic wave clock, and define “velocity” as Einstein does, it may be true that light has the same “velocity” in all 4-variable inertial coordinate systems that include an instance of the Einstein clock variable. But if this is true, it is a truth about the distorting effect of the spatially sloping Einstein clock variable, not a truth about the natural world.

VI. MAXWELL'S EQUATIONS

Maxwell's equations retain their mathematical form under the Lorentz transformation. Many consider this to be a virtue of Einstein's theory. It should be regarded as neither a virtue nor a vice, but merely as a mathematical fact, for the following two reasons.

First, the Lorentz transformation is applicable to Maxwell's equations only if every occurrence of t in them is interpreted as an instance of the Einstein clock variable. With t interpreted in this way, Maxwell's equations do not describe nature. The derivative of a variable with respect to an instance of the Einstein clock variable is not the variable's natural rate of change. It is slightly greater than or slightly less than the natural rate of change, depending on the motion of the array of clocks that defines the instance of the Einstein clock variable that is being used to compute the derivative. Thus, a mathematical relationship that many physicists find appealing is purchased at the price of artificiality and alienation from nature. For anyone whose purpose is to describe nature, this price is prohibitive.

Second, if my argument in Section I is sound, there is no good reason to insist on any symmetry condition between coordinate systems. Let the t in Maxwell's equations be interpreted in a way that aligns with the electromagnetic wave clock. Then it is possible that Maxwell's equations are strictly correct only in an inertial coordinate system in which a light pulse generates the same number of wave cycles on each leg of a round trip between two objects at rest in that coordinate system. In other coordinate systems, Maxwell's equations might be only approximately correct, with the approximation becoming gradually worse as the imbalance between the two legs of the round-trip increases. The approximation would be extremely good across a wide range of coordinate systems, making it difficult to tell the difference between approximate and strict correctness. It is also possible that Maxwell's equations are not strictly correct in any inertial coordinate system, but provide a very good approximation across a wide range of coordinate systems.

Either of these possibilities would be especially easy to accept if the coordinate-system-dependent equations can be derived from more fundamental facts that can be stated without using any coordinate system. I conjecture that this is generally if not always the case: Any truth that can be stated by using an object-anchored coordinate system can be deduced

from a conjunction of truths that can be stated without using any object-anchored coordinate system and the properties of object-anchored coordinate systems. Coordinate-system-dependent truths are never fundamental. If there are counterexamples to this conjecture, I would love to hear about them.

There are many interesting mathematical relationships which, as far as anyone knows, have no significance for the physical world. The mathematical relationship between Maxwell's equations and the Lorentz transformation could be one of these. The fact that this relationship was discovered by physicists is irrelevant. The mathematical groping of physicists, like the mathematical groping of mathematicians, should be expected to yield a mixture of mathematical discoveries with and without physical significance.

In his biography of Einstein, Albrecht Fölsing speculates as follows about the final hours before the birth of the special theory of relativity:

The fruitfulness of this exceedingly daring idea might have later struck Einstein at home, when he easily succeeded in deriving the “Lorentz-Fitzgerald contraction”—introduced by Lorentz into his theory as an independent hypothesis—from this modified time concept, without any further assumptions, and in thus obtaining a transformation of the local coordinates. As a skilled electrodynamicist he would then have examined the behavior of the Maxwell-Lorentz equations under these transformations. When it emerged in the course of his nocturnal calculations that these equations were invariant and that, moreover, the “Lorentz force,” introduced as an independent hypothesis into electron theory, also resulted readily from the transformation behavior, virtually everything was accomplished. Relativity principle and universal constancy of the velocity of light, Maxwellian theory and Lorentz transformations: everything came together in the most wonderful way, and the following morning Einstein jubilantly informed his friend Besso that he had “completely solved” the problem.¹³

I consider this a plausible reconstruction; the sequence of events in Einstein's mind might have been much like this. Fölsing's mistake is to frame this as an account of a major advance in humanity's quest to understand nature. It can and should be read very differently, as a description of a trap snapping shut. Every aspect of the pleasing mathematical mosaic that Fölsing describes depends on the artifice of a spatially sloping clock variable. The Lorentz transformation as derived by Einstein connects two 4-variable inertial coordinate systems that include instances of the spatially sloping Einstein clock variable. The relevance of this Lorentz transformation to Maxwell's equations depends on interpreting every t in Maxwell's equations as an instance of the spatially sloping Einstein clock variable. Einstein's replication of the algebra of the Lorentz-Fitzgerald contraction involves a fanciful procedure for measuring length that depends on the spatially sloping Einstein clock variable. The universal constancy of the velocity of light in Einstein's coordinate-

system-relative sense is the constant *levocity* of light, which is computed using the spatially sloping Einstein clock variable. The whole wonderful coming-together and attendant jubilation depends on using a spatially sloping clock variable without realizing that this is what you are doing.

The book *Old Physics for New* by Thomas E. Phipps Jr. contains an interesting discussion of that one of Maxwell's equations that corresponds to Faraday's law.¹⁴ Phipps argues for a version of this equation that is slightly more complicated than the "official" version and that, unlike the official version, does not satisfy the beloved Lorentz symmetry condition. He claims that the version he advocates covers all the cases that the official version covers, plus an important class of cases that fall under Faraday's law but which the official version does not cover. He further claims that many physics textbooks fudge the relevant mathematics in ways that obscure the difference between the two versions of this equation. In short, people sweep certain real phenomena and/or certain errors of reasoning under the rug in order to hold on to the Lorentz-symmetric version of this equation. I do not feel competent to judge the merit of Phipps's argument, but I commend it as a beautiful illustration of the relativity trap's potential for blocking progress in our understanding of the phenomena addressed by Maxwell's equations. If there is a way to improve these equations as a result of which they no longer satisfy the Lorentz symmetry condition, little attention will be paid to it as long as the Lorentz symmetry condition and the underlying Einstein clock variable retain their insidious mystique.

VII. CRITICIZING THE SPECIAL THEORY OF RELATIVITY

This final section makes a variety of points that provide valuable context for the criticism of the special theory of relativity that I have presented in this paper.

First, I occasionally come across the remark that the only legitimate way to criticize the special theory of relativity is to produce experimental results that do not match the theory's predictions. This is not true of any theory. Theories aspire to explain, not just to predict. If a theory is to provide a correct explanation of what its supporters claim to predict with it, it must have internal integrity. A theory that generates predictions by means of false propositions, fallacious reasoning, and covert arbitrary steps does not explain anything. For explanations, one must look elsewhere. There is no room here to discuss the many problems with the ways in which physicists have generated predictions in the name of the special theory of relativity, or the experiments that are said to "confirm" these predictions.

Second, there are other valid criticisms of the internal integrity of the special theory of relativity, which are logically independent of the criticism I have presented in this paper. Ironically, one reason for the persistence of this theory is that it contains multiple errors that are woven together in such a way that they cover for each other. This paper focuses on one fundamental error; it is not a comprehensive survey of everything that is wrong with the theory.

Third, although the special theory of relativity contains other significant errors, very little of the theory survives

the criticism presented in this paper. I have shown how Einstein's procedure for coordinating clocks produces a clock variable that has a nonzero spatial gradient, and I have shown how all the sensational claims of the theory flow from the use of this spatially sloping clock variable, when combined with the branding maneuver of re-defining the mother-tongue vocabulary of "time," "synchronous," "simultaneous," and "velocity." There are a few statements commonly classified as elements of the special theory of relativity that survive this criticism. For example, I have said nothing that contradicts the claim that nothing can travel faster than light. Likewise, I have said nothing that contradicts the bare claim (as opposed to Einstein's derivation of it) that mass and energy are interconvertible in accordance with the formula $E = mc^2$. There may be other claims that are commonly considered part of the special theory of relativity that survive the criticism presented in this paper, but the heart and soul of the theory does not.

Fourth, many published criticisms of the special theory of relativity have defects of their own. Some are poorly written and difficult to make sense of. Some exhibit a misunderstanding of the theory that they are ostensibly criticizing. Some contain errors of reasoning. Some combine criticism of the special theory of relativity with advocacy of dubious rival theories. It is unfortunate that so many bad criticisms exist, because a bad criticism of a bad theory tends to make the theory look good. But such is life. The point to remember is that a correct criticism is correct no matter how many bad criticisms have been published.

Fifth and finally, I think it is important for critics of the special theory of relativity to study one another's work and try to build an informed consensus as to what the errors in the theory really are. Powerful forces maintain the grip of this deeply flawed theory on our society. These forces include authoritative-sounding books, university curricula, vested career interests, prejudicial journal policies, the Einstein-was-a-genius mythology, and good old peer pressure and herd psychology. As a practical matter, a widening critical consensus is needed in order to loosen the theory's iron grip.

I will now make a modest attempt at consensus building by explaining an interesting connection between this paper and a paper that makes a different criticism of Einstein's method of coordinating clocks. That other paper is "Einstein's Third Postulate" by Wolfgang Engelhardt.¹⁵ It focuses on a section of the book *The Evolution of Physics*, which was co-authored by Einstein and Leopold Infeld and published in 1938, 33 years after the paper in which Einstein pioneered the theory.¹⁶ In his paper, Engelhardt makes the following three related claims:

1. The section of *The Evolution of Physics* that he discusses contains a logical contradiction.
2. A certain statement in Einstein's 1905 paper, which Engelhardt dubs Einstein's third postulate, is false.
3. The falsehood of Einstein's third postulate is the source of the logical contradiction in *The Evolution of Physics*.

In my view, Engelhardt is partly right and partly wrong. There is indeed a logical contradiction in *The Evolution*

of *Physic*s. However, the statement that Engelhardt calls Einstein's third postulate seems plausible to me, and in any case that statement is not the source of the contradiction in *The Evolution of Physic*s. Rather, the contradiction that Engelhardt identifies in *The Evolution of Physic*s is rooted in the problem that I have brought to light in this paper. This fact ties the two papers together in an interesting way.

You can follow my discussion of Engelhardt's paper and the relevant section of *The Evolution of Physic*s without having read either of those items. Of course, you will need to read those items in order to judge for yourself whether what I say here is correct.

Engelhardt quotes the statement that he calls Einstein's third postulate as follows:

We assume that this definition of synchronism is free from contradictions and possible for any number of points;¹⁵

This is the first part of a sentence which I will quote in full. The sentence is part of the passage on coordinating clocks that I quoted in Section II of this paper, where a different English translation is used:

We assume that it is possible for this definition of synchronism to be free of contradictions, and to be so for arbitrarily many points, and that the following relations are therefore generally valid:

1. If the clock in B is synchronous with the clock in A, then the clock in A is synchronous with the clock in B.
2. If the clock in A is synchronous with the clock in B as well as with the clock in C, then the clocks in B and C are also synchronous relative to each other.⁸

Here is the German original:

Wir nehmen an, dass diese Definition der Synchronismus in widerspruchsfreier Weise möglich sei, und zwar für beliebig viele Punkte, dass also allgemein die Beziehungen gelten:

1. Wenn die Uhr in B synchron mit der Uhr in A läuft, so läuft die Uhr in A synchron mit der Uhr in B.
2. Wenn die Uhr in A sowohl mit der Uhr in B als auch mit der Uhr in C synchron läuft, so laufen auch die Uhren in B und C synchron relativ zueinander.¹⁷

This sentence may be open to different interpretations, so I will spell out its meaning as I understand it. As I explained in Section II, in order to define an instance of the Einstein clock variable for a particular inertial coordinate system, one must coordinate all the clocks in a spatially distributed array using the method that Einstein specifies. Suppose there are n clocks. Then there are $\frac{n(n-1)}{2}$ pairs of clocks to which you could apply Einstein's clock-coordination procedure. How-

ever, in order to coordinate all the clocks, it suffices to apply the procedure to $n - 1$ pairs. There are many ways to do this. You can select one clock to serve as the "home clock" and coordinate each of the other $n - 1$ clocks with it. Or you can proceed in a series, first coordinating clock A with clock B, then clock B with clock C, and so on through all the clocks, in any order. Other approaches are possible. After Einstein's procedure has been applied to a sufficient set of $n - 1$ pairs, those $n - 1$ pairs will have been coordinated directly while all the other pairs—the vast majority of the $\frac{n(n-1)}{2}$ total number—will have been coordinated indirectly as a result of the direct coordination steps. The sentence in question claims that if you now directly coordinate a pair of clocks that has been coordinated indirectly, no change in the clock settings will be called for; coordinating two clocks directly always gives the same result as coordinating them indirectly.

I am not sure whether this claim is true, but it seems plausible to me. I note, moreover, that Engelhardt does not try to show that Einstein's clock-coordination procedure is internally contradictory. Nor does he try to show how an inconsistency in Einstein's clock-coordination procedure would produce the logical contradiction that he sees in *The Evolution of Physic*s. It is really just a conjecture on Engelhardt's part that the statement he calls Einstein's third postulate is the culprit. The case for the innocence of that statement is further strengthened by the explanation that follows of where the contradiction in *The Evolution of Physic*s really comes from.

Here is what is going on in the section of *The Evolution of Physic*s that Engelhardt criticizes. Einstein and Infeld wrote this book for a general audience. In order to cater to such an audience, they simplified their presentation of certain matters as compared with Einstein's 1905 paper.

One simplification is the use of a different procedure for coordinating clocks. The procedures are similar, and the authors obviously believe that they give the same result, but it is not easy to verify this because the procedure in *The Evolution of Physic*s is described in a brief and folksy manner whereas the procedure in Einstein's 1905 paper is described at greater length and with algebraic notation. I will assume that the two clock-coordination procedures give the same result.

The authors make another simplification, which alters the theory in a nontrivial way. The presentation in *The Evolution of Physic*s does not declare any definitions. The authors do not say they are *defining* the words "synchronous" and "simultaneous;" they simply use these words. They present the Einstein clock-coordination procedure as a way to synchronize clocks, thereby encouraging readers to conjoin that procedure with their entire mother-tongue understanding of the words "synchronize" and "synchronous." In effect, the authors assert a theorem that the Einstein clock-coordination procedure results in an array of clocks that are mother-tongue synchronous, and they do this without acknowledging that they are doing it, without attempting to prove such a theorem, and without even saying what they think being synchronous in the mother-tongue sense amounts to. Were the 1938 authors oblivious to the fact that they were leaving out

the 1905 definitions? Were they aware that they were leaving out the 1905 definitions but thought that this made no difference? Nothing in the text suggests answers to these intriguing questions. In Section II, I warned that the practice of giving a novel technical definition to a word that already has a mother-tongue meaning creates a risk of confusing the two meanings. Here in *The Evolution of Physics* the authors go beyond creating a risk of confusion. They make the confusion happen.

The authors are themselves victims of this confusion. They draw a set of diagrams in which a coordinate system furnished with “synchronous” clocks is depicted with three clocks in different spatial locations that have identically configured faces representing identical numerical readings. *These clock faces depict a clock variable that has a zero spatial gradient.* The authors draw their diagrams this way, I presume, because such a picture is part of their mother-tongue understanding of the word “synchronous.” The trouble is that the diagrams are said to depict clocks that have been coordinated by Einstein’s clock-coordination procedure, which produces a clock variable that has a *nonzero* spatial gradient, as I showed in Section III. It risked confusion, but was not untrue, for Einstein to decree that clocks coordinated in his way are “synchronous” *by definition*. But it is untrue to claim, using either words or diagrams, that clocks coordinated in Einstein’s way define a clock variable that has a zero spatial gradient. This falsehood is the result of equivocating on the word “synchronous” and thus conjoining, without supporting argument, the clock-coordination procedure that is Einstein’s 1905 definition of this word with all of this word’s mother-tongue associations.

The contradiction that Engelhardt identifies is an immediate consequence of this false depiction of clocks that have been coordinated by Einstein’s procedure. In my terminology, Engelhardt’s claim is that two sets of clocks belonging to different inertial coordinate systems cannot each have the zero-spatial-gradient property depicted in the diagrams in *The Evolution of Physics* and also be interrelated by the Lorentz transformation. Engelhardt is plainly right about this. The Lorentz transformation goes hand-in-hand with clock-variable instances that have nonzero spatial gradients. If you lay down a definition that permits clocks that define a spatially sloping variable to be “synchronous,” then two sets of “synchronous” clocks can be interrelated by the Lorentz transformation. But if “synchronous” clocks are understood in a mother-tongue way as defining a zero-spatial-gradient variable, then clock synchrony and the Lorentz transformation are logical oil and water.

One could argue that the contradiction Engelhardt identifies in *The Evolution of Physics* is not strictly speaking a property of the special theory of relativity, because this contradiction is not present in the pioneering 1905 paper. The

contradiction is created in the 1938 popularization when the authors dispense with the 1905 definitions and simply use the word “synchronous” in its mother-tongue sense. However, in redefining the word “synchronous,” which suggests a zero spatial gradient, to brand a variable that does not in fact have that characteristic, the 1905 paper dances right on the brink of this contradiction. It is a very small step from the latently contradictory 1905 branding-by-definition to the full-blown contradiction in the 1938 popularization. Indeed, this step is so small that the author of the 1905 paper took it in 1938 apparently unaware that he was doing so.

In conclusion, I want to stress that the criticism I have presented in this paper is not about logical contradictions. I have shown that the special theory of relativity rests on the use of a clock variable that is not aligned with the most accurate of clocks, the electromagnetic wave clock. With the exception of one very special case, instances of this Einstein clock variable have nonzero spatial gradients. They are artificial constructs that do not have any business in the project of describing nature.

¹A. Einstein, *Ideas and Opinions* (Three Rivers Press, New York, 1954), p. 230.

²A. Einstein, *Ideas and Opinions* (Three Rivers Press, New York, 1954), p. 229.

³A. Einstein, *The Collected Papers of Albert Einstein, Vol. 2: Writings 1900–1909* (English translation supplement, Princeton University Press, Princeton, NJ), p. 141 (<https://einsteinpapers.press.princeton.edu/papers>).

⁴A. Einstein, *Phys. Today* 35, 45 (1982).

⁵A. Einstein, *Ideas and Opinions* (Three Rivers Press, New York, 1954), p. 246.

⁶A. Einstein, *The Collected Papers of Albert Einstein, Vol. 2: Writings 1900–1909* (English translation supplement, Princeton University Press, Princeton, NJ), p. 141.

⁷A. Einstein, *The Collected Papers of Albert Einstein, Vol. 2: Writings 1900–1909* (English translation supplement, Princeton University Press, Princeton, NJ), p. 141.

⁸A. Einstein, *The Collected Papers of Albert Einstein, Vol. 2: Writings 1900–1909* (English translation supplement, Princeton University Press, Princeton, NJ), pp. 142–143.

⁹A. Einstein, *Relativity: The Special and the General Theory* (Mockingbird Classics Publishing, Fairhope, AL, 2015), p. 26.

¹⁰H. Minkowski, *Space and Time: Minkowski’s Papers on Relativity* (Minkowski Institute Press, Montreal, QC, Canada, 2012), p. 111.

¹¹A. Einstein, *The Collected Papers of Albert Einstein, Vol. 2: Writings 1900–1909* (English translation supplement, Princeton University Press, Princeton, NJ), pp. 143–144.

¹²A. Einstein, *Relativity: The Special and the General Theory* (Mockingbird Classics Publishing, Fairhope, AL, 2015), pp. 25–26.

¹³A. Fölsing, *Albert Einstein: A Biography* (Penguin Books, New York, 1997), p. 177.

¹⁴T. E. Phipps, Jr., *Old Physics for New: A Worldview Alternative to Einstein’s Relativity Theory*, 2nd ed. (Apeiron, Montreal, QC, Canada, 2012).

¹⁵W. Engelhardt, *Phys. Essays* 29, 513 (2016).

¹⁶A. Einstein and L. Infeld, *The Evolution of Physics* (Simon & Schuster, New York, 2007), pp. 177–184.

¹⁷A. Einstein, *The Collected Papers of Albert Einstein, Vol. 2: The Swiss Years: Writings 1900–1909* (Princeton University Press, Princeton, NJ), p. 279.